

## **University of Florida**

# Optimization, Robustness and Equilibrium Modeling for the Florida Smart Grid

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**Description:** The purpose of this research is to develop models and algorithms for optimal design and functioning of the nation's next generation power transmission and distribution system that will incorporate the new realities of the grid. Our goal is to create innovative real time capabilities for 1) optimal functioning of renewable energy sources (location, charging, discharging of batteries, etc.), 2) detecting and preventing instabilities and outages, and 3) operating models including generalized Nash equilibrium.

**Budget:** \$30,000 **Universities:** UF

#### **Progress Summary**

The project develops a game theoretic approach for electricity market participants with storage devices. With electricity prices changing continuously over day storage devices can be used to reduce electricity consumption during peak-hours as well as reducing electricity prices, carbon emissions and peak transmission loads. However, if everyone shifts their demand toward a period when electricity is cheaper, that will have an inevitable effect on electricity price and will not lead to significant reduction of a peak demand but rather shift it for another period of the day. The goal is to develop a model for "smart batteries" – a plan for charging and discharging batteries in such a way that every participant will enjoy the maximal possible gain. The model developed in the project formulates a Nash equilibrium problem and propose extensions for generalized Nash equilibrium. In the simplest case, our model presents a Nash equilibrium problem with quadratic cost functions. It is attacked with several methods recently developed.

### Funds leveraged/new partnerships created:

Steffen Rebennack, PhD, Assistant Professor Colorado School of Mines Division of Economics and Business 816 15th Street Golden, CO, 80501, USA

Neng Fan, PhD Sandia National Laboratories

#### **2011 Annual Report**

We consider a problem of micro-storage management where household communities have common batteries installed and can buy electricity from the grid for home use, for charging the battery, or can use battery for the house when the current electricity price is high.





















The objective is to develop a model such that every agent minimizes the cost for electricity and battery running cost. The model describes a Nash equilibrium problem and proposes extensions for generalized Nash equilibrium. While the theory of the generalized Nash equilibrium is well developed, its computation is a challenge. The difficulty stems from the fact that the Nash equilibrium is a fixed point of an appropriate mapping, and its calculation goes beyond the optimization theory. In the simplest case, our model presents a Nash equilibrium problem with quadratic cost functions. It is attacked with several methods recently developed. With electricity prices changing continuously over day storage devices can be used to reduce electricity consumption during peak-hours as well as reducing electricity prices, carbon emissions and peak transmission loads. However, if everyone shifts their demand toward a period when electricity is cheaper, that will have an inevitable effect on electricity price and will not lead to significant reduction of a peak demand but rather shift it for another period of the day.

### **Assumptions:**

- There are several communities present and together they can affect electricity price by changing electricity demand.
- Every community shares a common battery.

The first assumption leads to Nash equilibrium problem, i.e. the solution of each agent problem depends on the rest agents. The second assumption leads to a generalized Nash equilibrium problem, which is much more difficult. Both models are new and difficult to solve. The fact that several communities work on Nash equilibrium problem (without knowing each to other) shows the importance and applicability of this model where non-cooperative equilibrium is sought.

- 0.1. **Agents.** There is a set of agents (customers) A who want to minimize their electricity cost. The agents form communities i, which are presented in a set I. An agent has a load profile  $l_t^{ia}$ , which shows the demand for electricity at any time moment  $t \in T$  not considering the battery charging/discharging. The sum of load profiles among the agents of all communities will define the total electricity demand at time moment  $t \in T$ :  $d_t = \sum_i \sum_a l_t^{ia}$ . Every agent has an access to a common battery within a community i with the following parameters:
  - total capacity  $e^i$ ,
  - efficiency  $\alpha^i$ ,
  - running cost  $c^i$ ,

For minimizing the cost, an agent a can change their storage profile  $b_t^{ia}$ ,  $\forall t \in T$ :  $-b_-^i + \sum_{A/a} b_{t-}^i \le b_t^{ia} \le b_+^i - \sum_{A/a} b_{t+}^i$ , where  $b_-^i$  is a discharging capacity of the battery and  $b_+^i$  is charging capacity of the battery. Clearly, the charging profile for an agent a at time moment t is a difference between the amount charged and discharged:  $b_t^{ia} = b_t^{ia+} - b_t^{ia-}$ . Summing up over all the agents we get the net storage profile:  $b_t = \sum_i \sum_a b_t^{ia}$ .

0.2. **Market.** Total amount of electricity bought from the market at time moment t is  $q_t = d_t + b_t$ . The market price of the electricity is defined by supply curve of that market, which is assumed to be a nondecreasing:  $p_t = s_t(q_t)$  and each agent will pay  $p_t \cdot (l_t^{ia} + b_t^{ia})$ . Total cost of electricity for all the agents will be  $p_t \cdot q_t$ .



















Set of communities

Set of agents within community i

Set of time periods

load profile – demand not considering battery use

 $d_t = \sum_i \sum_a l_t^{ia}$  load profile – demand not condition  $l_t^{ia}$  Total electricity demand at t

 $e^i$  battery total capacity

battery efficiency, if q is charged then  $\alpha^i q$  is discharged

 $c^i$  battery running cost

storage profile – amount of electricity charged/discharged at time moment t by an agent a in a community i

Discharging capacity of a battery i

Charging capacity of a battery i

Amount of electricity charged at time period t

Amount of electricity discharged at time moment t

Amount of electricity that can be discharged in battery i at time moment t

Amount of electricity that can be charged to battery i at time moment t

Price of electricity at time moment t

#### The Model:

Every agent minimizes the cost for electricity and battery running cost:

(1) 
$$cost^{ia}(b^{ia}) = \sum_{t} \left( p_t \cdot (l_t^{ia} + b_t^{ia}) + c^i \cdot b_t^{ia+} \right)$$

storage profile:

(2) 
$$b_t^{ia} = b_t^{ia+} - b_t^{ia-}, \ \forall i \in I, \ a \in A, \ t \in T,$$

total daily charging can not exceed battery capacity:

(3) 
$$\sum_{a} \sum_{t} b_t^{ia+} \le e^i, \ \forall i \in I,$$

battery efficiency constraints:

(4) 
$$\sum_{a} \sum_{t} b_{t}^{ia-} = \sum_{a} \sum_{t} \alpha^{i} b_{t}^{ia+}, \ \forall i \in I,$$

charging profile feasibility constraints:

(5) 
$$\sum_{a} b_{t}^{ia+} \leq q_{t}^{i+}, \ \forall i \in I, \ t \in T,$$
(6) 
$$\sum_{a} b_{t}^{ia-} \leq q_{t}^{i-}, \ \forall i \in I, \ t \in T,$$

(6) 
$$\sum_{a} b_t^{ia-} \le q_t^{i-}, \ \forall i \in I, , t \in T,$$

(7) 
$$q_t^{i-} = \alpha^i \left( q_0^{i+} + \sum_a \sum_{k=1}^{t-1} (b_k^{ia+} - b_k^{ia-} / \alpha^i) \right), \ \forall i \in I, t \in T,$$

(8) 
$$q_t^{i+} = e^i - \left( q_0^{i+} + \sum_{a} \sum_{k=1}^{t-1} (b_k^{ia+} - b_k^{ia-} / \alpha^i) \right), \ \forall i \in I, t \in T,$$

electricity reselling is not allowed:

(9) 
$$l_t^{ia} \ge b_t^{ia}, \ \forall i \in I, \ a \in A, \ t \in T.$$





















#### **Activities:**

Organized conference (Organizer Panos Pardalos)

Systems and Optimization Aspects of Smart Grid Challenges

April 28-30, 2011 Gainesville, Florida, USA

Presented talk: "Game Theoretic Approach for Micro-storage Management in the Smart Grid", by Pando Georgiev, Alexey Sorokin, Marco Carvalho and Panos Pardalos.

Accepted talk at the INFORMS conference, November 16 "Nash Equilibrium Model for Micro-storage Management in the Smart Grid" by Alexey Sorokin, Pando Georgiev, Marco Carvalho and Panos Pardalos.

Working towards to publish the results in this talk in a journal paper.

Ongoing work on data mining in energy for detecting and preventing instabilities and outages of the power grid.

#### **Edited books:**

<u>Handbook of Networks in Power Systems I</u> co-editors: Alexey Sorokin, Steffen Rebennack, Panos Pardalos, Niko Iliadis, Mario Pereira, Springer, (2011).

<u>Handbook of Networks in Power Systems II</u> co-editors: Alexey Sorokin, Steffen Rebennack, Panos Pardalos, Niko Iliadis, Mario Pereira, Springer, (2011).

















