A Thermodynamic Perspective on Energy Efficiency

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RÉFLEXIONS

SUR LA

PUISSANCE MOTRICE

DU FEU

ET

SUR LES MACHINES

PROPRES A DÉVELOPPER CETTE PUISSANCE.

PAR S. CARNOT,

ANCIEN ÉLÈVE DE L'ÉCOLE POLYTECHNIQUE.

A PARIS,

CHEZ BACHELIER, LIBRAIRE,

QUAI DES AUGUSTINS, N°. 55.

1824.

Nicolas Léonard Sadi Carnot. 1796-1832

The question has often been raised whether the motive power of heat* is unbounded, whether the possible improvements in steam-engines have an assignable limit,—a limit which the nature of things will not allow to be passed by any means whatever; or whether, on the contrary, these improvements may be carried on indefinitely.

It is natural to ask here this curious and important question: Is the motive power of heat invariable in quantity, or does it vary with the agent employed to realize it as the intermediary substance, selected as the subject of action of the heat?

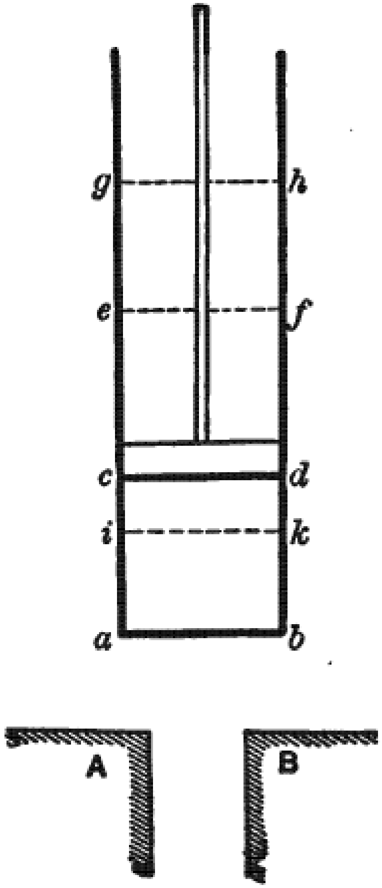
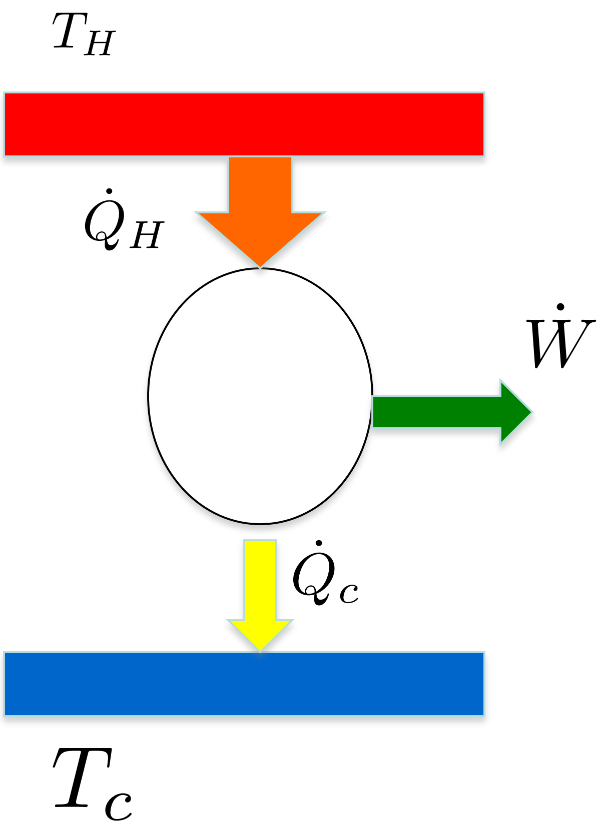
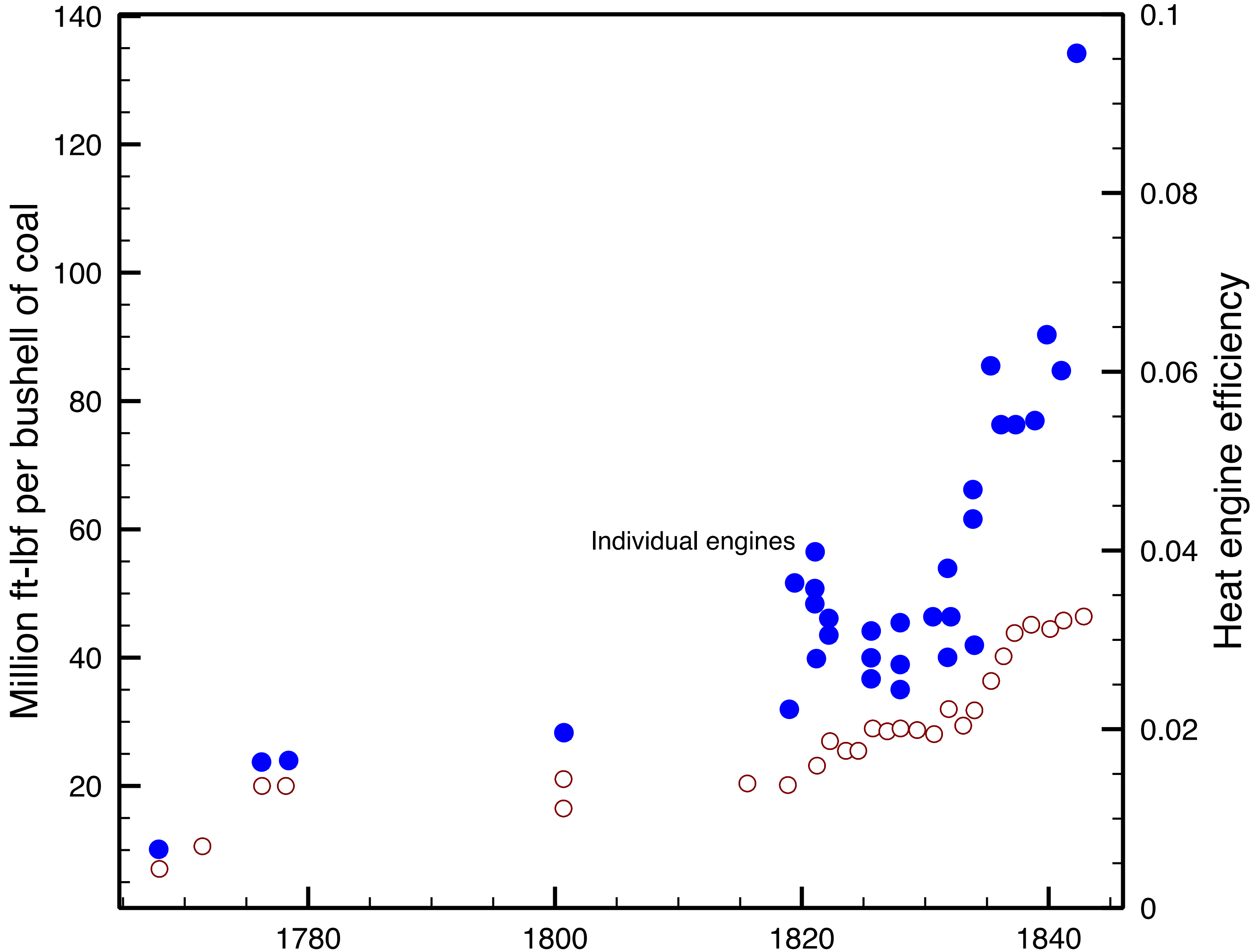


FIG. 1.



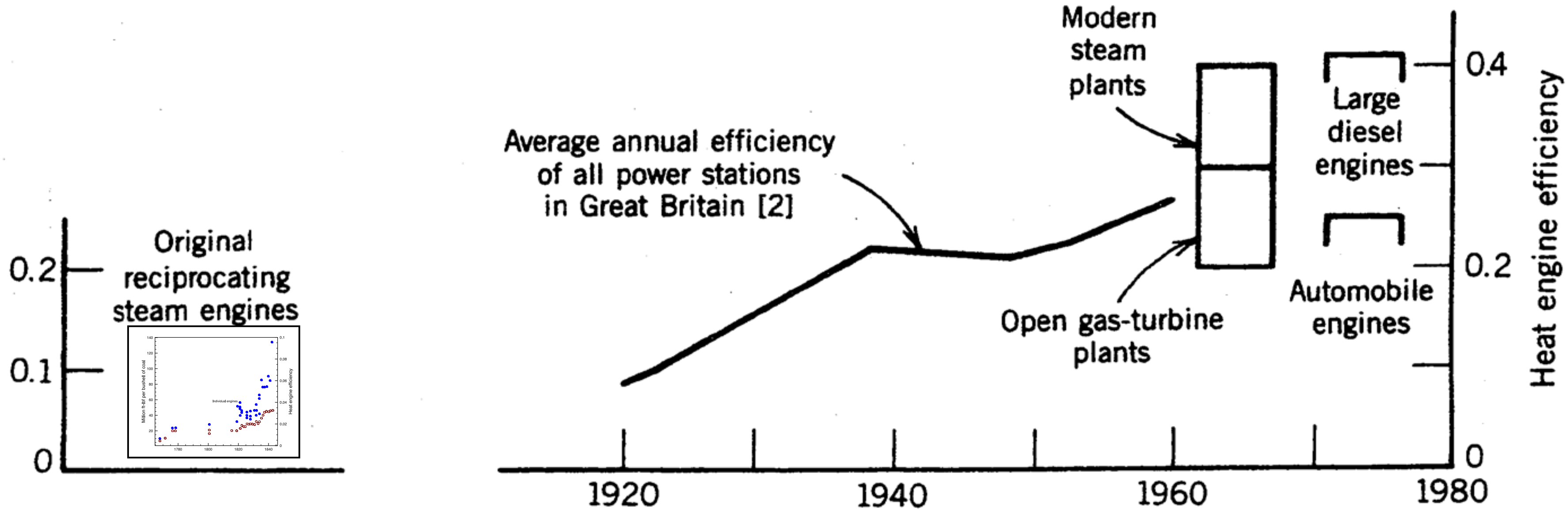
Thermal engine

$$\eta = \frac{\dot{W}}{\dot{Q}_H}$$



Engine Efficiencies in times of Carnot

Drawn after Bejan –AET and Cardwell
"From Watt to Clausius"

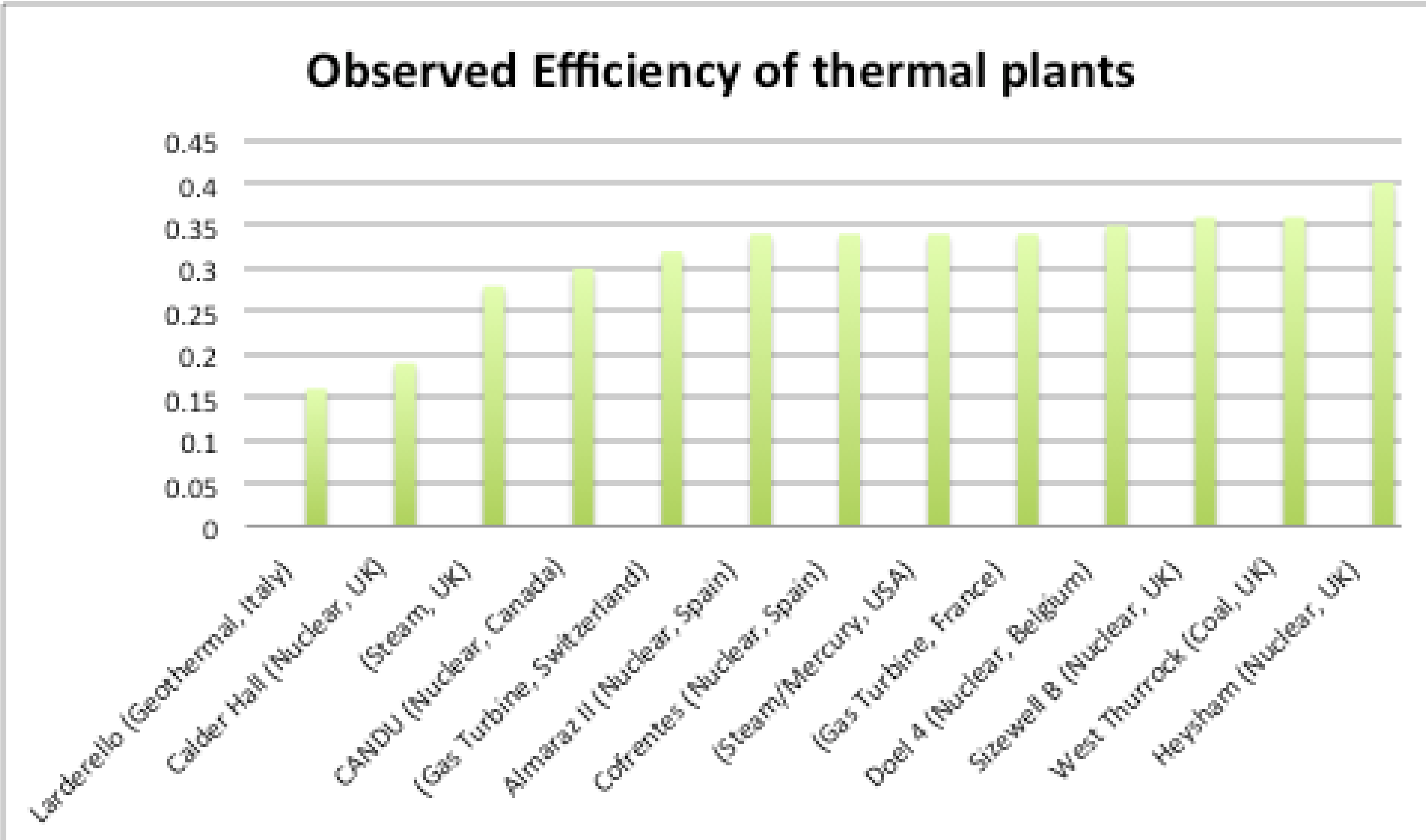


0.2
0.1
0
Original
reciprocating
steam engines
Average annual efficiency
of all power stations
in Great Britain [2]
Modern
steam
plants
Open gas-turbine
plants
Large
diesel
engines
Automobile
engines
0.4
0.2
0
Heat engine efficiency
1920
1940
1960
1980

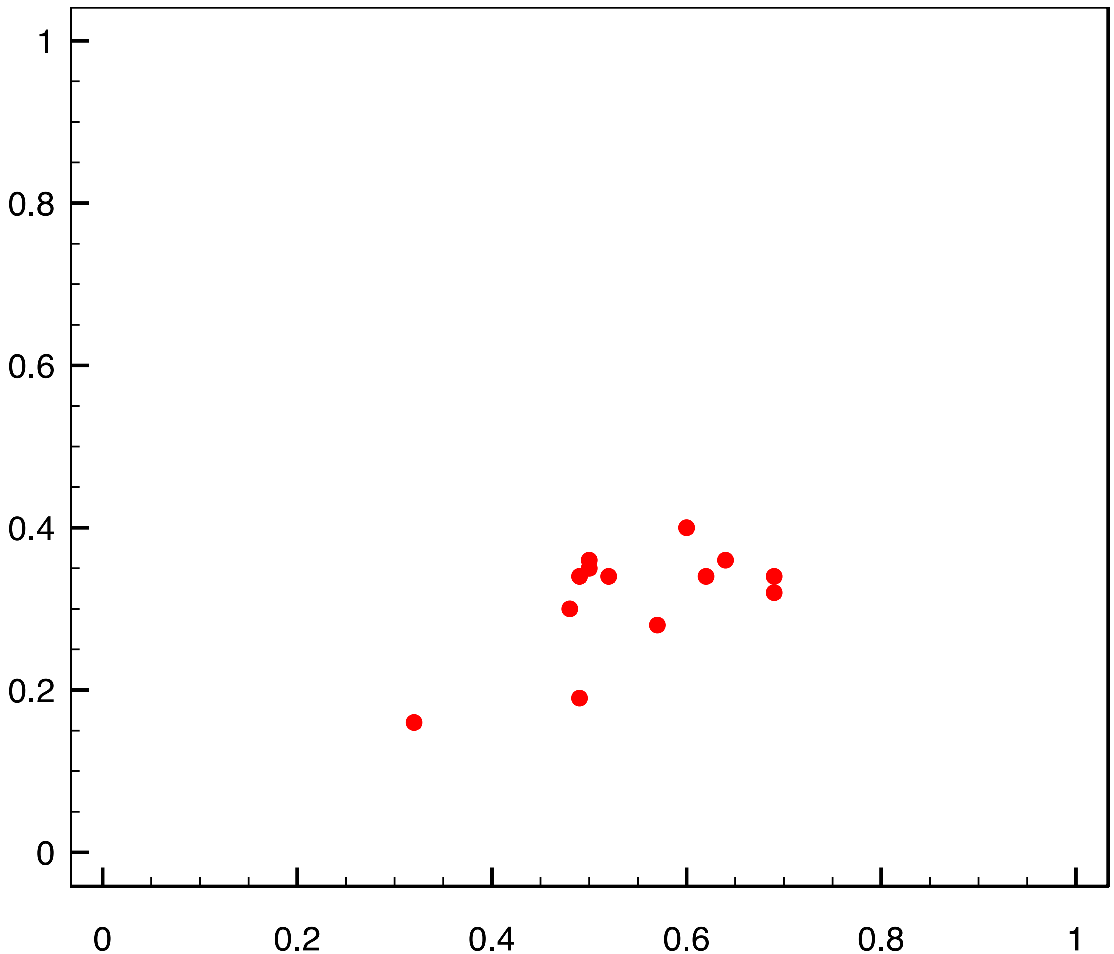
TABLE I. Theoretical bounds and observed efficiency η_{obs} of thermal plants.

Plant	$T_h(K)$	$T_c(K)$	η_C	η_{obs}
Doel 4 (Nuclear, Belgium) [6]	566	283	0.5	0.35
Almaraz II (Nuclear, Spain) [6]	600	290	0.52	0.34
Sizewell B (Nuclear, UK) [6]	581	288	0.5	0.36
Cofrentes (Nuclear, Spain) [6]	562	289	0.49	0.34
Heysham (Nuclear, UK) [6]	727	288	0.60	0.40
West Thurrock (Coal, UK) [1]	838	298	0.64	0.36
CANDU (Nuclear, Canada) [1]	573	298	0.48	0.30
Larderello (Geothermal, Italy)[1]	523	353	0.32	0.16
Calder Hall (Nuclear, UK) [6]	583	298	0.49	0.19
(Steam/Mercury,USA) [6]	783	298	0.62	0.34
(Steam, UK) [6]	698	298	0.57	0.28
(Gas Turbine, Switzerland) [6]	963	298	0.69	0.32
(Gas Turbine, France) [6]	953	298	0.69	0.34

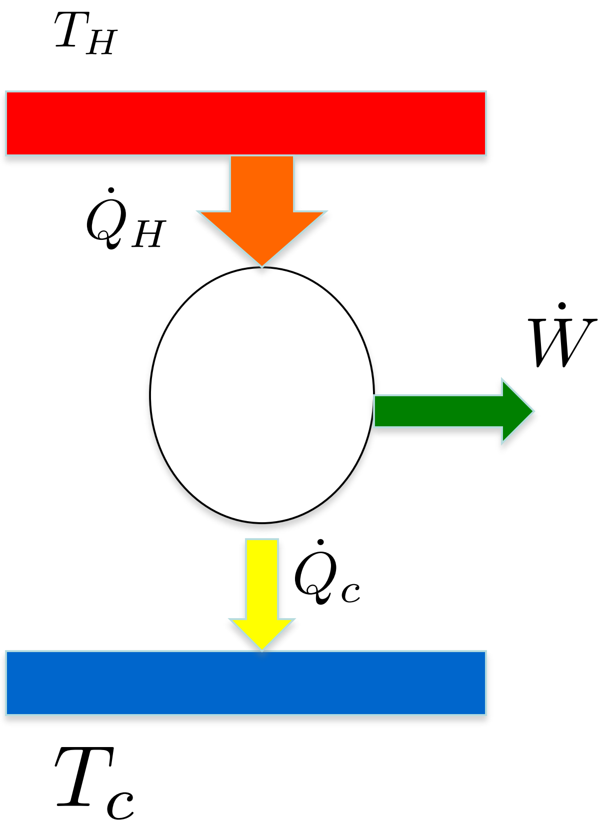
Sources: Bejan (AET); and H.B. Callen (T)

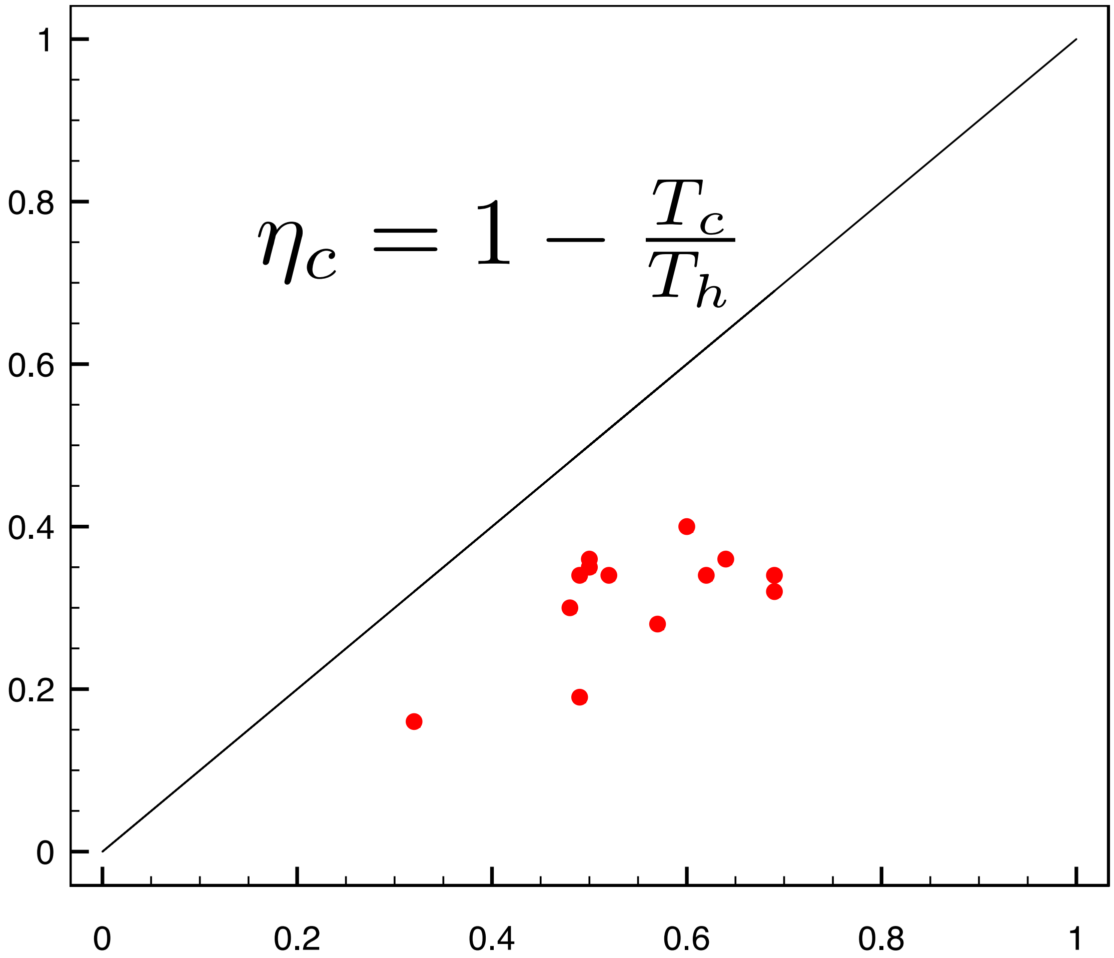

Observed Efficiency of thermal plants
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Larderello (Geothermal, Italy)
Calder Hall (Nuclear, UK)
(Steam, UK)
CANDU (Nuclear, Canada)
(Gas Turbine, Switzerland)
Almaraz II (Nuclear, Spain)
Cofrentes (Nuclear, Spain)
(Steam/Mercury, USA)
(Gas Turbine, France)
Doel 4 (Nuclear, Belgium)
Sizewell B (Nuclear, UK)
West Thurrock (Coal, UK)
Heysham (Nuclear, UK)

Observed efficiency



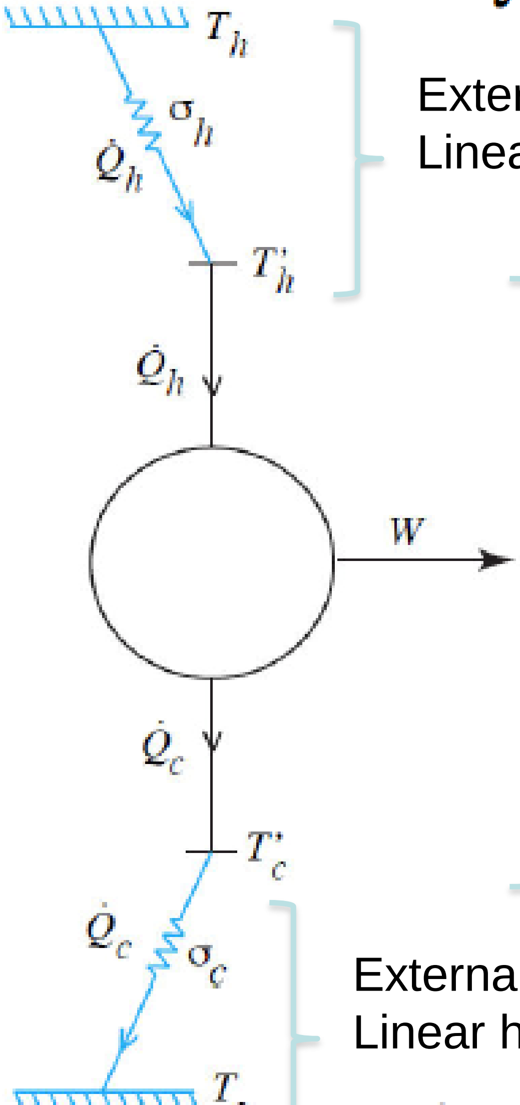
$$\eta_C = 1 - \frac{T_c}{T_H}$$




$\eta_c = 1 - \frac{T_c}{T_h}$
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1

Endoreversible model

$$\dot{Q}_h = \sigma_\mathrm{h}\left(T_\mathrm{h} - T'_\mathrm{h}\right)$$



External irreversibility
Linear heat transfer model

endoreversible

External irreversibility
Linear heat transfer model

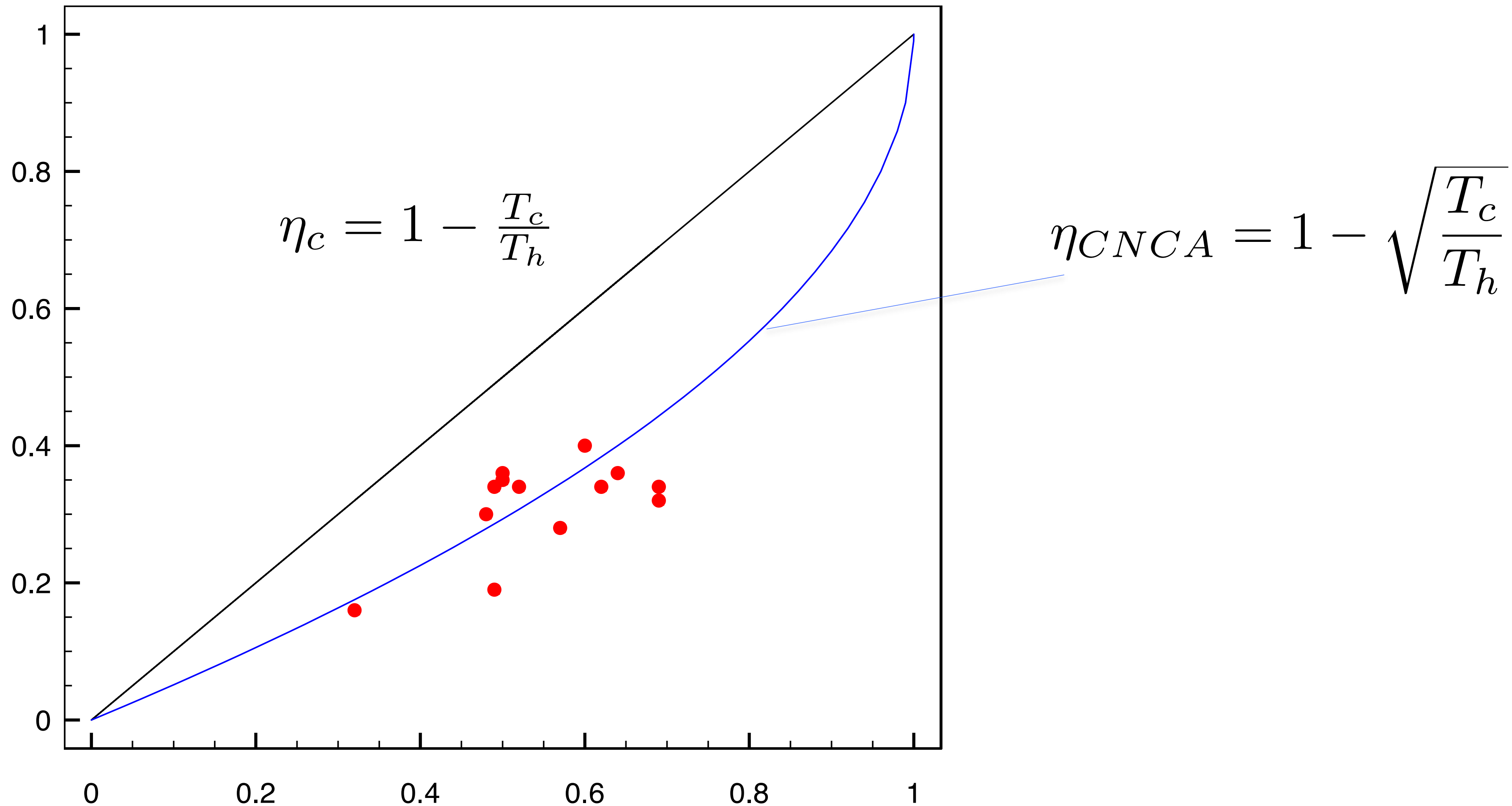
$$\dot{Q}_c = \sigma_\mathrm{c}\left(T_\mathrm{c} - T'_\mathrm{c}\right)$$

Chambadal (1957)
Novikov (1958)
Curzon & Alhborn (1975)

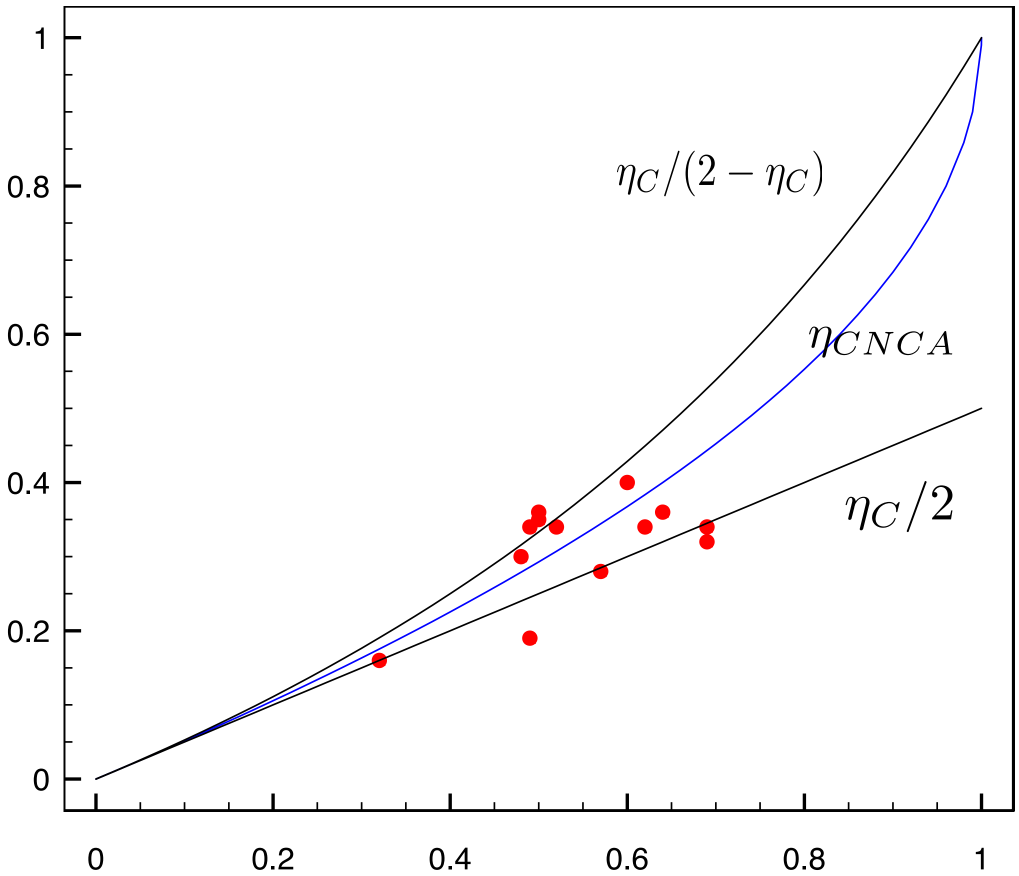
$$\dot{W}_{\max} = \sigma_h T_h \frac{\left(1 - \sqrt{\tau}\right)^2}{1 + \sigma_{hc}}$$

$$\tau = \frac{T_c}{T_h} \qquad \sigma_{hc} = \frac{\sigma_h}{\sigma_c}$$

$$\eta_{CNCA} = 1 - \sqrt{\frac{T_c}{T_h}}$$



$\eta_c = 1 - \frac{T_c}{T_h}$
$\eta_{CNCA} = 1 - \sqrt{\frac{T_c}{T_h}}$
0
0.2
0.4
0.6
0.8
1



Recent studies suggest that at maximum power

$$\frac{\eta_C}{2} <= \eta <= \frac{\eta_C}{2-\eta_C}$$

$$\eta_{CNCA} = \eta_{CA} = 1 - \sqrt{\tau} = 1 - \sqrt{1-\eta_C}$$

$$\approx \frac{\eta_C}{2} + \frac{\eta_C^2}{8} + \ldots$$

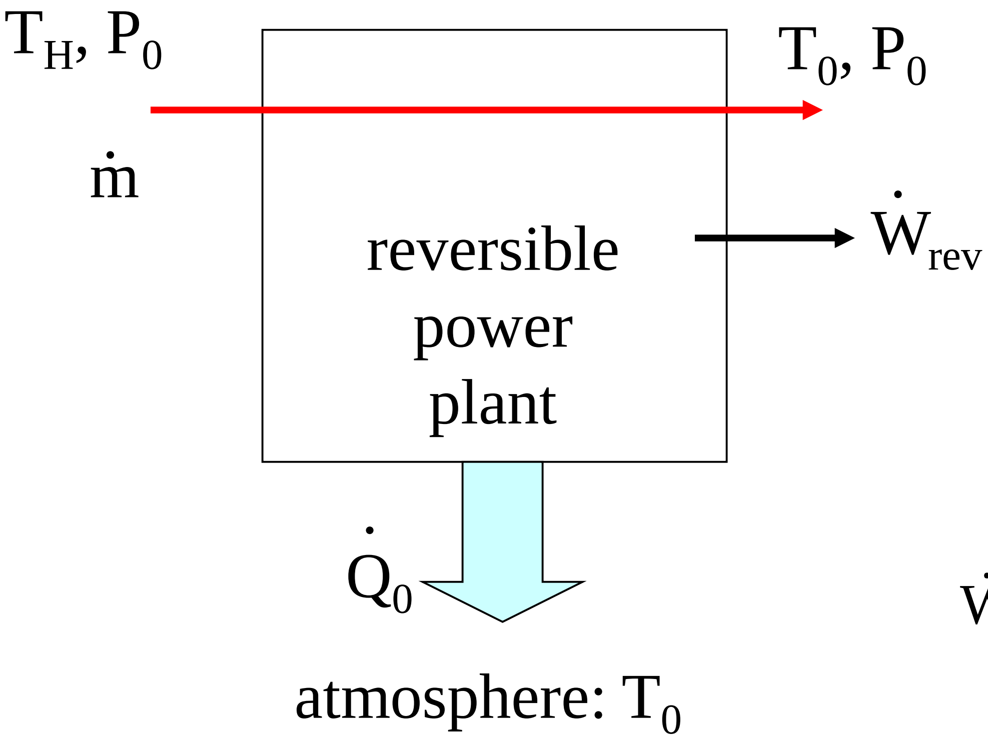
Taylor expansion

J.C. Ordóñez

It is natural to ask here this curious and important question : Is the motive power of heat invariable in quantity, or does it vary with the agent employed to realize it as the intermediary substance, selected as the subject of action of the heat ?

What is the maximum power that can be extracted from a hot stream? What are realistic limits for this power?

1. Power extraction:



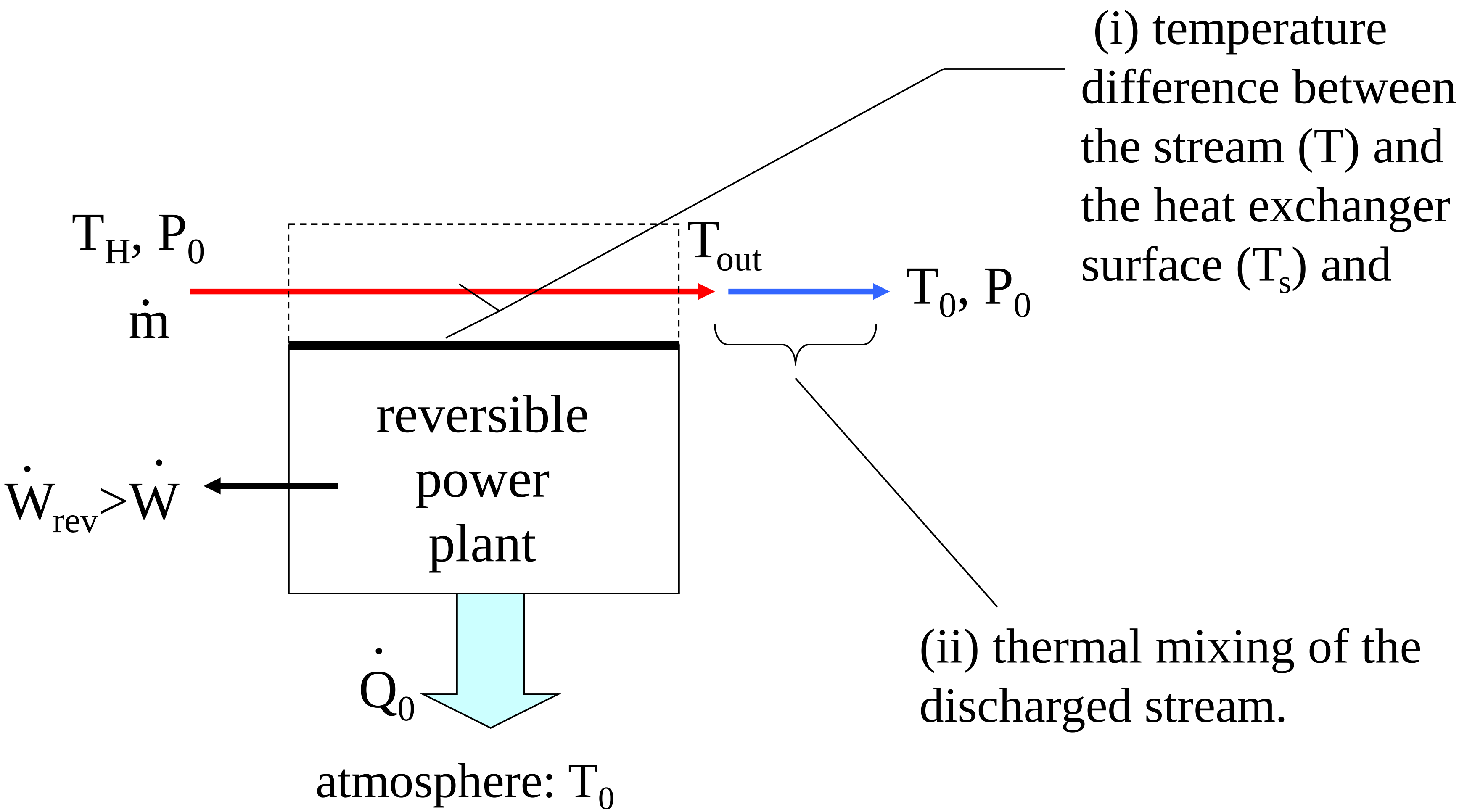
If the stream interacts only with the atmospheric temperature reservoir (T_0), the maximum power that can be extracted is the ***flow exergy***:

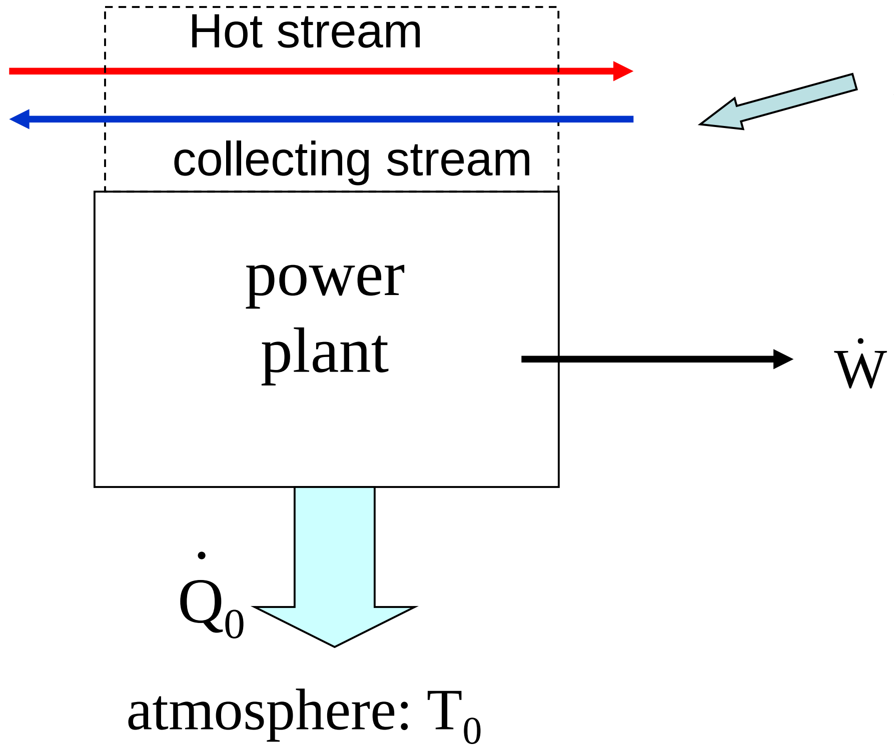
$$\dot{W}_{rev} = \dot{m} c_p T_0 \left(\frac{T_H}{T_0} - 1 - \ln \frac{T_H}{T_0} \right)$$

(ideal gas)

The actual power will always be lower than $\dot{W}_{rev}$ because of the irreversibilities of the heat transfer between the hot stream and the rest of the power plant.

The heat transfer irreversibility is due to:

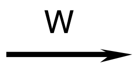




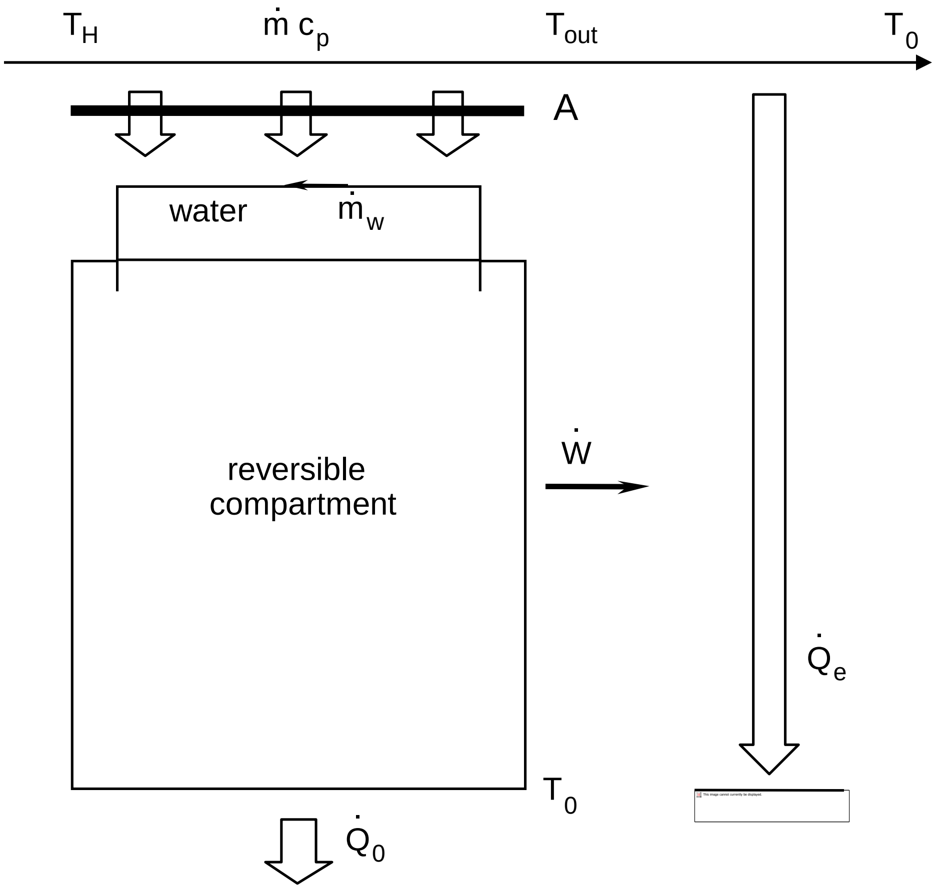
We use a second stream to collect the power from the hot stream. This second stream is the working fluid of the power producing device

The case where the collecting stream is single phase was studied by Bejan and Errera, 1998.

Power extraction from a hot stream



Entropy generation analysis



$$\dot{W} = \dot{m}\left(e_{x,2} - e_{x,1}\right)$$

$$\dot{W} = \dot{m}\left(h_H - h_0\right) - \dot{Q}_0 - \dot{Q}_e$$

$$\dot{S}_{gen} = \frac{\dot{Q}_0 + \dot{Q}_e}{T_0} + \dot{m}\left(s_0 - s_H\right) \geq 0$$

$$\dot{W} = \dot{m}e_{x,H} - T_0\dot{S}_{gen}$$

$$e_{x,H} = \left(h_H - T_0 s_H\right) - \left(h_0 - T_0 s_0\right)$$

Now for the heat exchanger, and external cooling alone:

$$\dot{m}\left(h_H - h_0\right) - \dot{m}_w\left(h_2 - h_1\right) - \dot{Q}_e = 0$$

$$\dot{S}_{gen} = \dot{m}\left(s_0 - s_H\right) + \dot{m}_w\left(s_2 - s_1\right) + \frac{\dot{Q}_e}{T_0} \geq 0$$

$$To\dot{S}_{gen} = \dot{m}e_{x,H} - \dot{m}_w\left(e_{x,2} - e_{x,1}\right)$$

We can maximize the power output using:

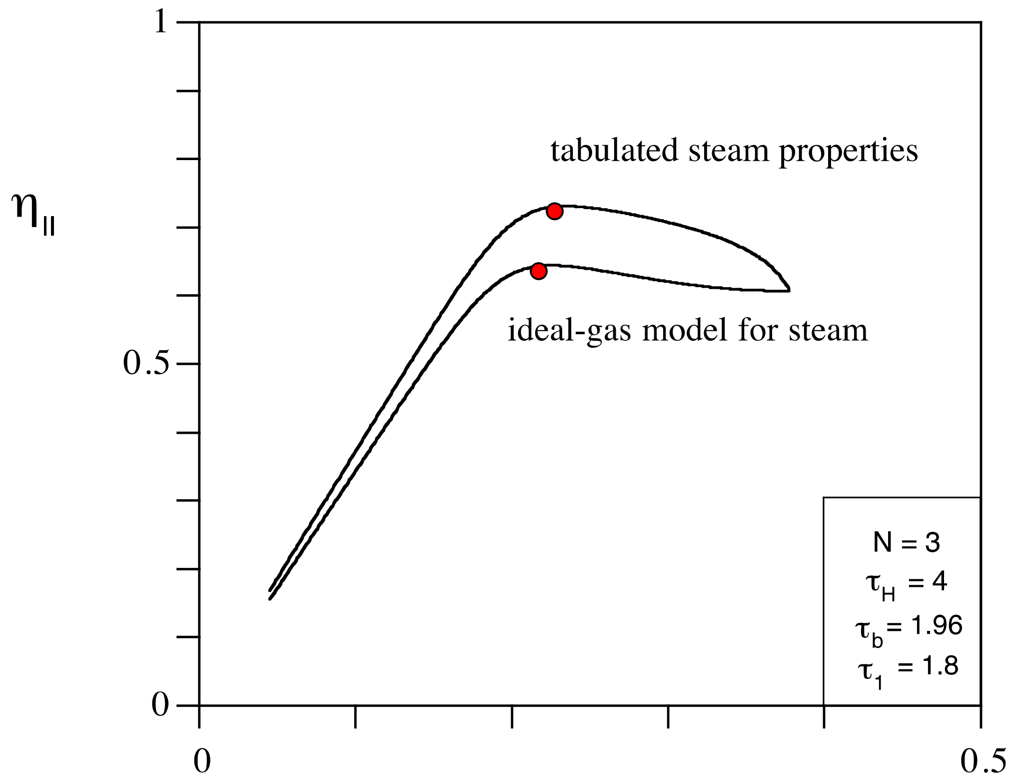
$$\dot{W} = \dot{m}e_{x,H} - T_0 \underbrace{\dot{S}_{gen}}_{\text{minimize}}$$

or directly using

$$\dot{W} = \dot{m}\left(e_{x,2} - e_{x,1}\right)$$

In <u>dimensionless form</u>:

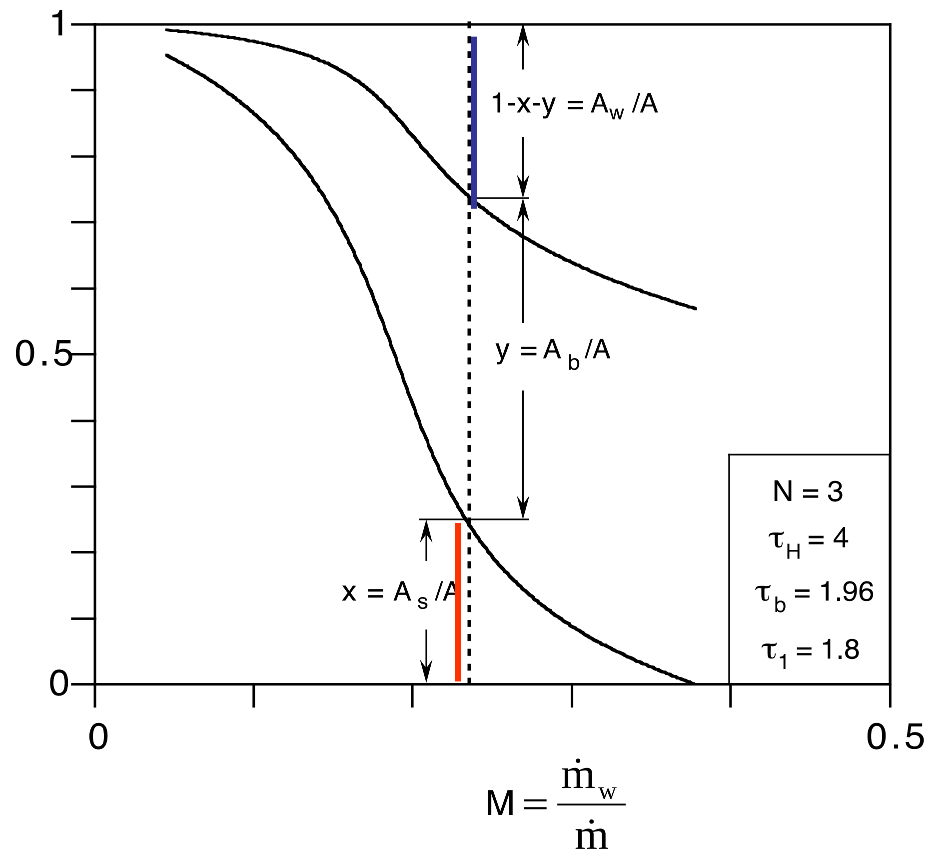
$$\eta_{II} = \frac{\dot{W}}{\dot{m}e_{x,H}} = \frac{\dot{m}_w\left(e_{x,2} - e_{x,1}\right)}{\dot{m}e_{x,H}}$$

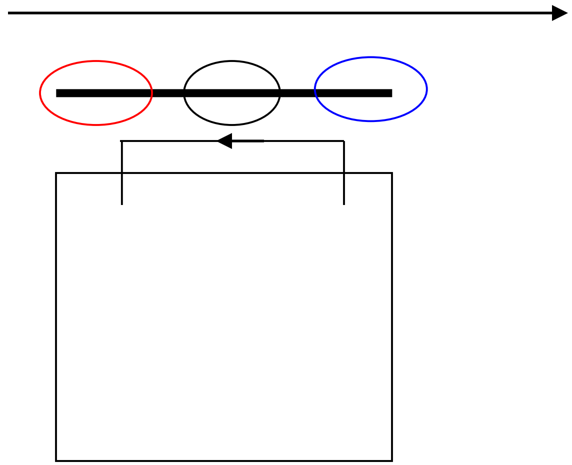


$$M = \frac{\dot{m}_w}{\dot{m}}$$

[IJHMT Vargas, Ordonez and Bejan, 1999]

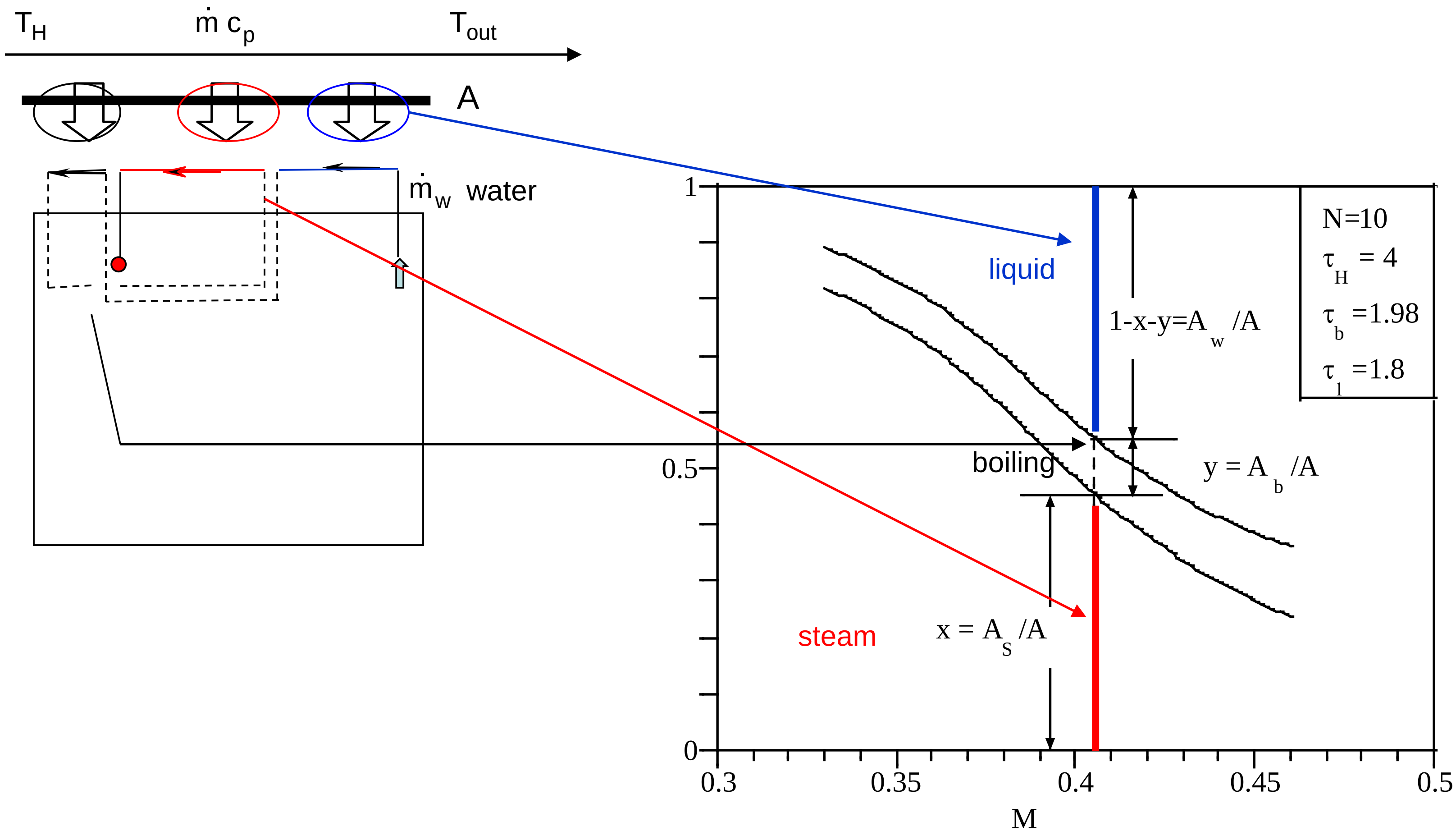
Maximization of the second law efficiency by selecting the mass flow rate of the water stream





Optimal system structure

Effect of the mass flow rate on the allocation of area among the sections of the heat exchanger



Effect of the mass flow rate on the allocation of area among the sections of the heat exchanger

Concluding remarks

- Sadi Carnot, laid out the foundations of thermodynamics exploring limits of operation of thermal engines that lead to maximum power.
- An interesting question that can be asked is why do we observe a pattern in efficiency at maximum power?
    - This maybe linked to symmetry in physical laws

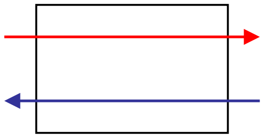
Concluding remarks

- Chambadal, Novikov, Curzon and Ahlborn derived efficiency expression that predicts well performance of thermal plants.

- The Novikov–Chambadal-Curzon-Ahlborn expression has been derived in different context:
  - Classical thermodynamics
  - Endoreversible thermodynamics (finite times, finite sizes)
  - Linear Irreversible Thermodynamics

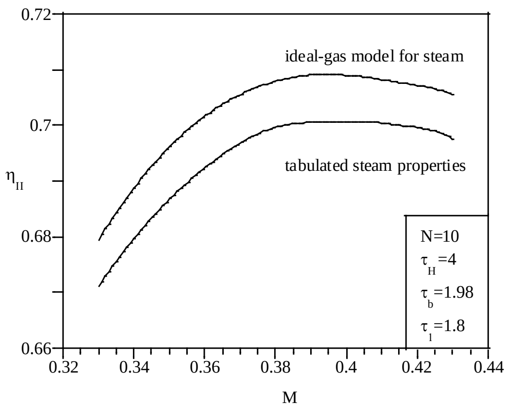
Concluding remarks:

• The **extraction of power and refrigeration** from a hot stream **can be maximized** by properly **matching** the stream with a receiving stream of cold fluid, across a finite-size heat transfer area

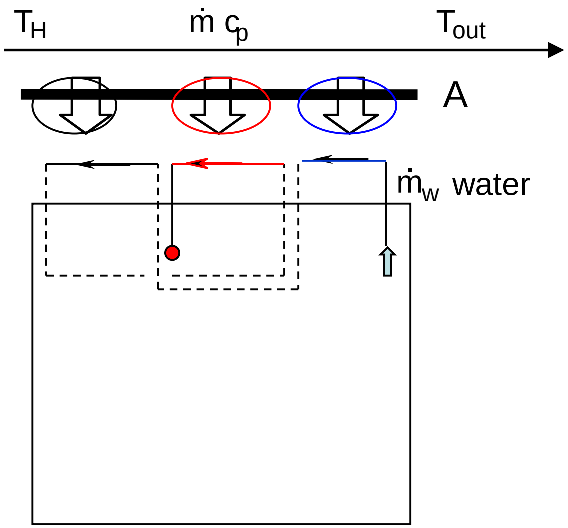


counterflow

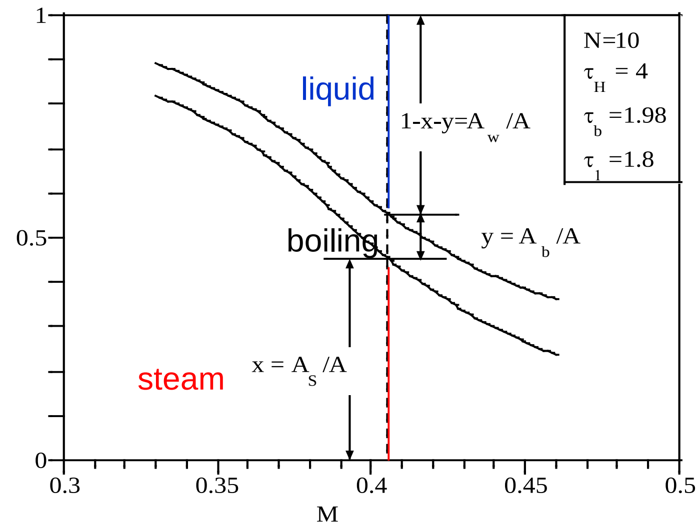
+



Optimal mass flow rate ratio



There is an associated optimal allocation of heat exchanger inventory:



Concluding remarks

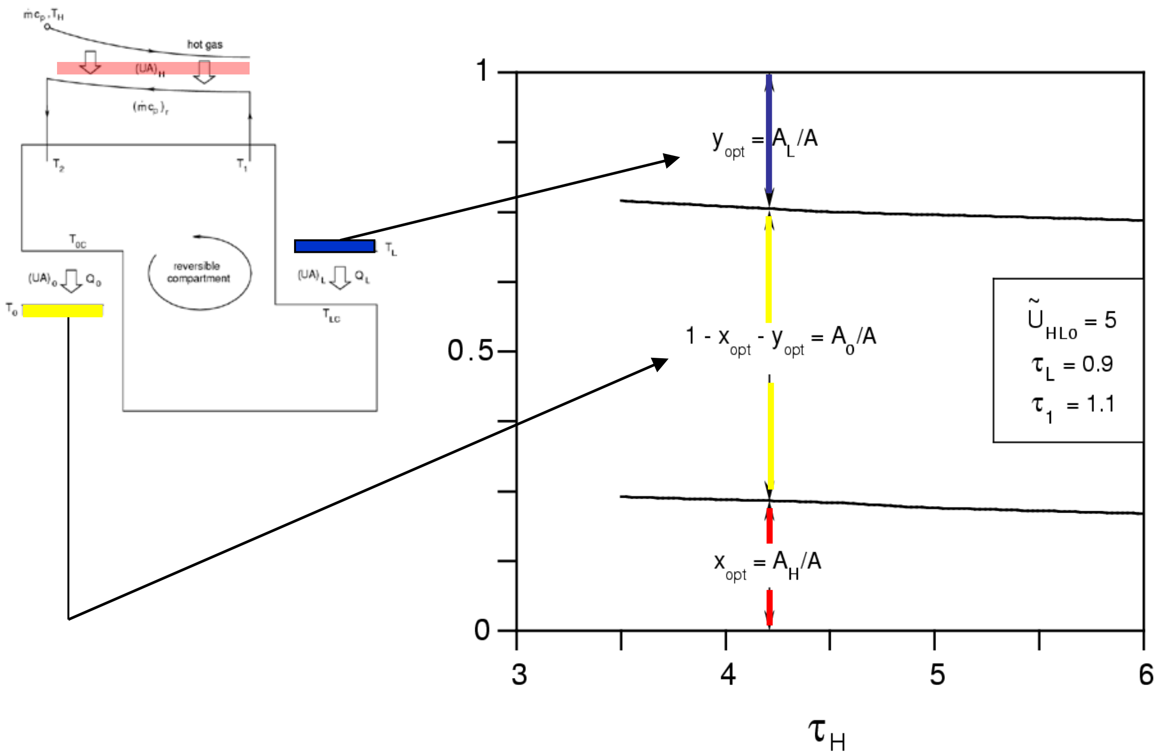
- System structure appears as a result of optimization -> maximization of flow access
- Constructal Design: “Generation of architecture under global constraints”

Thank you!

Acknowledgements:

- Prof. S.A. Sherif
- Professors A. Bejan, J.V. C. Vargas and C. Harman
- The comments of the reviewers of this paper are greatly appreciated.
- Center for Advanced Power Systems at Florida State University.

System structure appears as a result of optimization

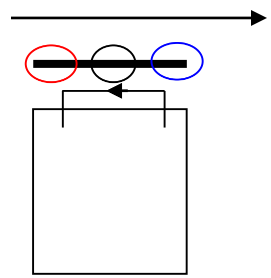


Constructal Design: “Generation of architecture under global contraints”

Concluding remarks:

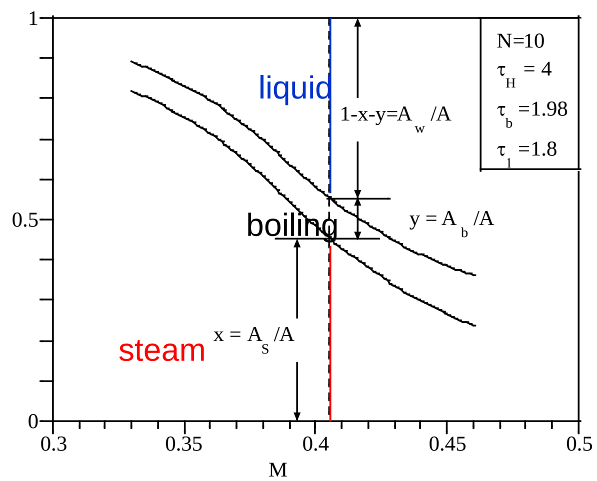
How does the optimal area allocation appears in practice?

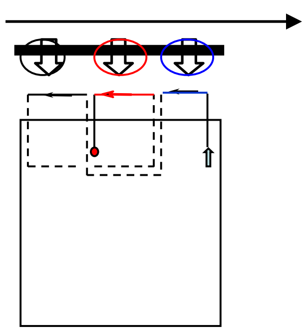
a) One counterflow heat exchanger, three sections: The three sections rearrange themselves



b) Three sections, boiling in contact with hottest gases:

Here a 'morphing' heat exchanger is needed

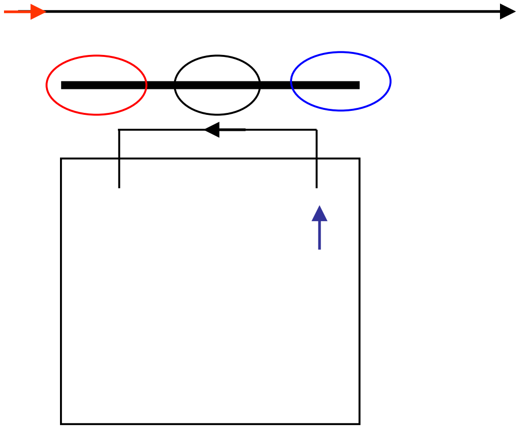


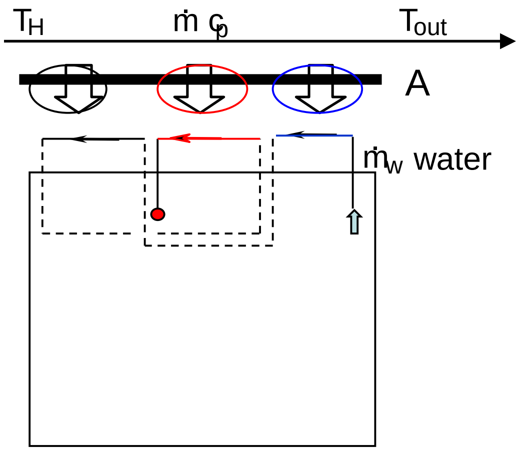


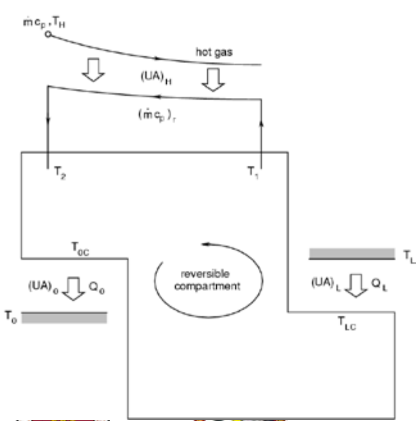
System structure appears as a result of optimization (Constructal theory)

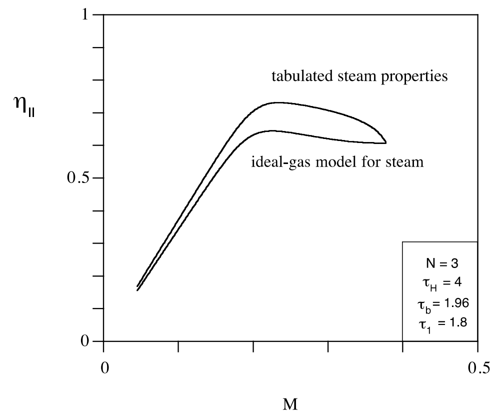
Concluding remarks:

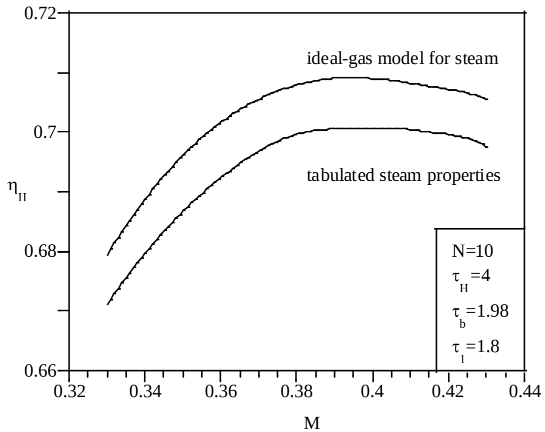
Optimal HT area allocation

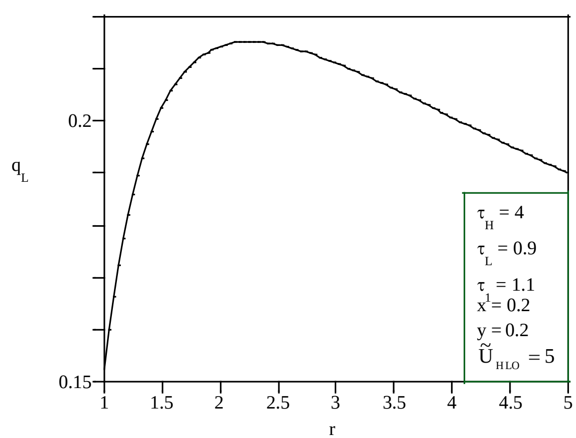


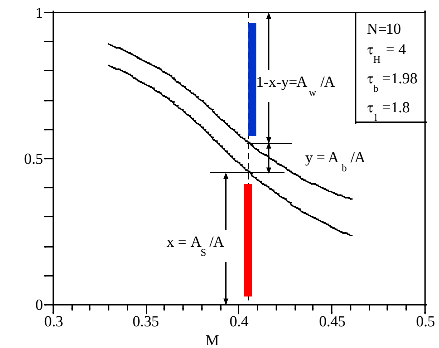












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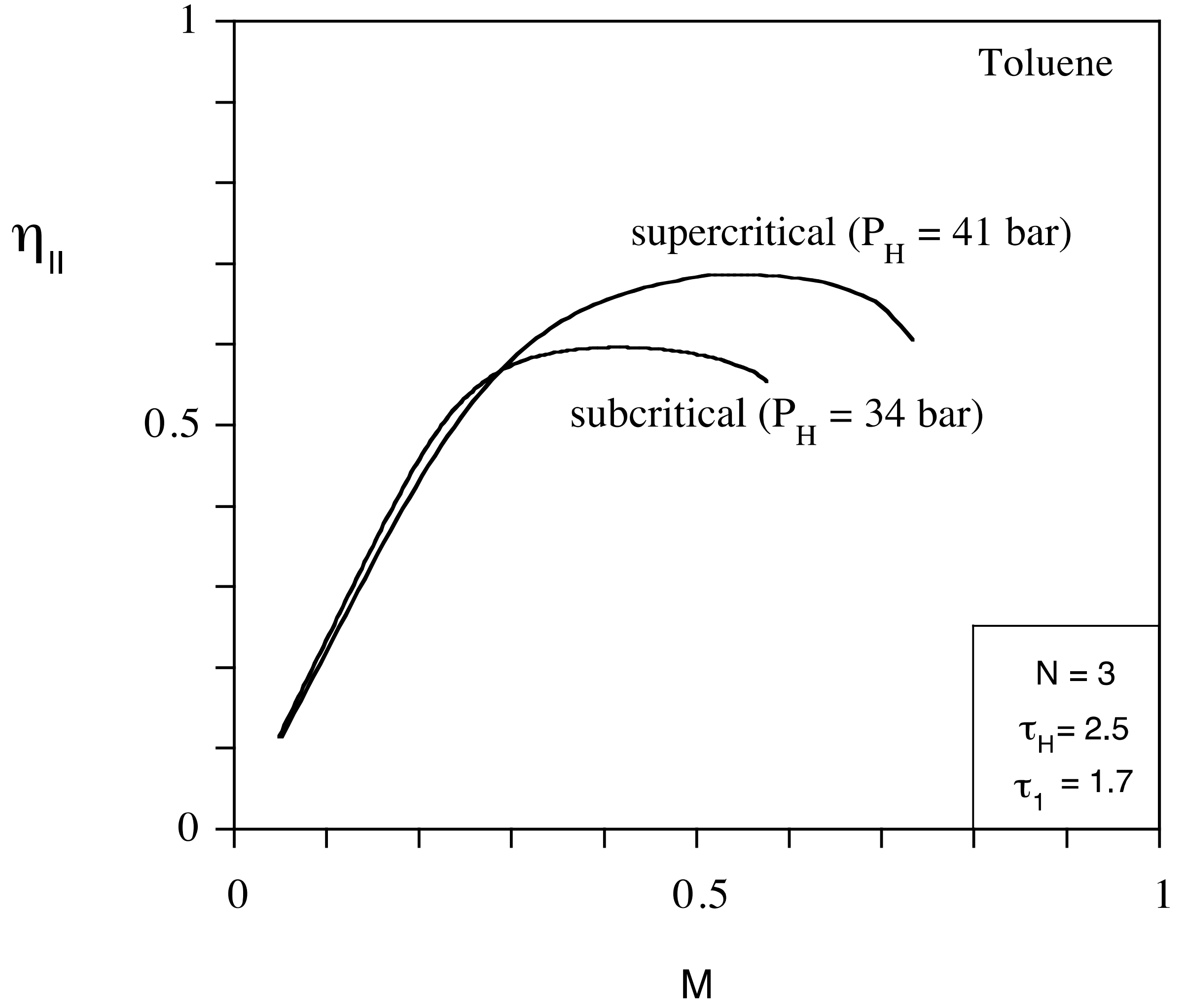
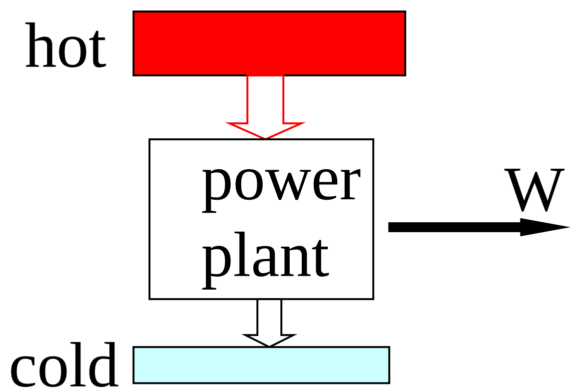


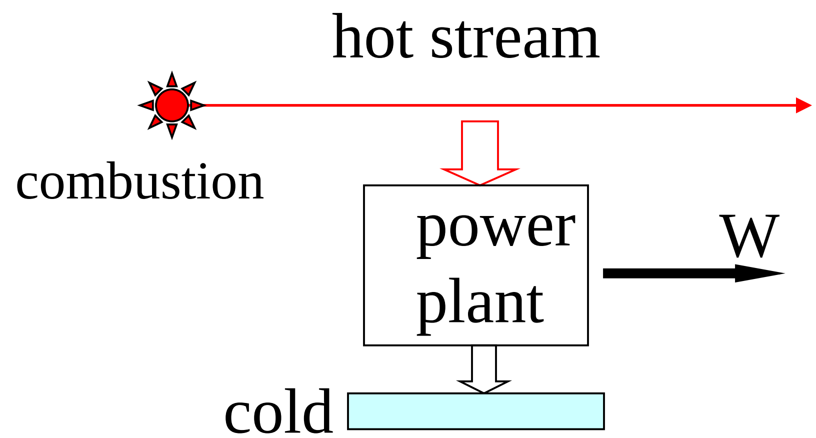
Maximization of the second law efficiency using Toluene as working fluid

Maximum power from a hot stream

- In engineering thermodynamics it is usually assumed that the heat that drives a power plant is already available from a hot temperature reservoir.



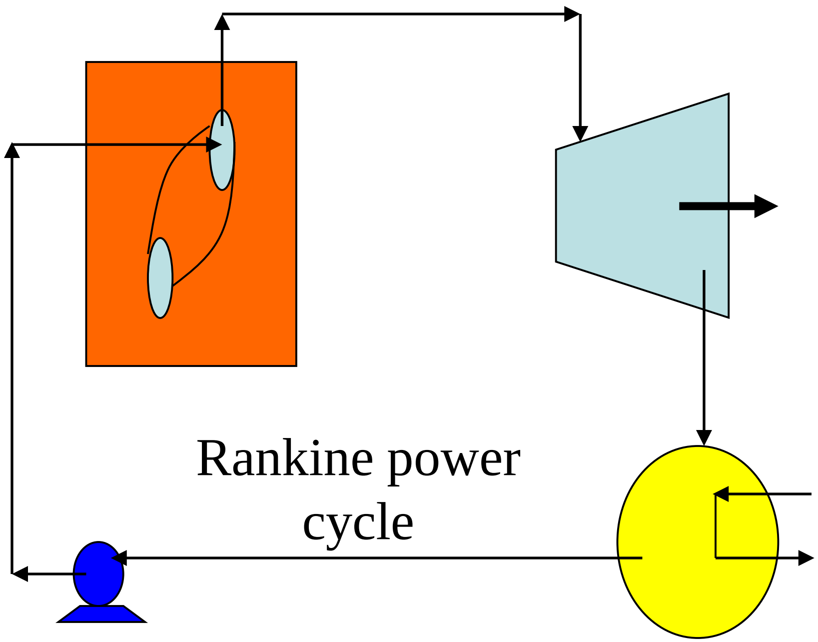
•In most applications a fuel is burn, and a hot stream becomes the input to the power plant.



- What is the maximum power that can be extracted from a hot stream?

Thermodynamic optimization methodology:

- Power and refrigeration systems are assemblies of streams and hardware (components).



- The size of the hardware is constrained.

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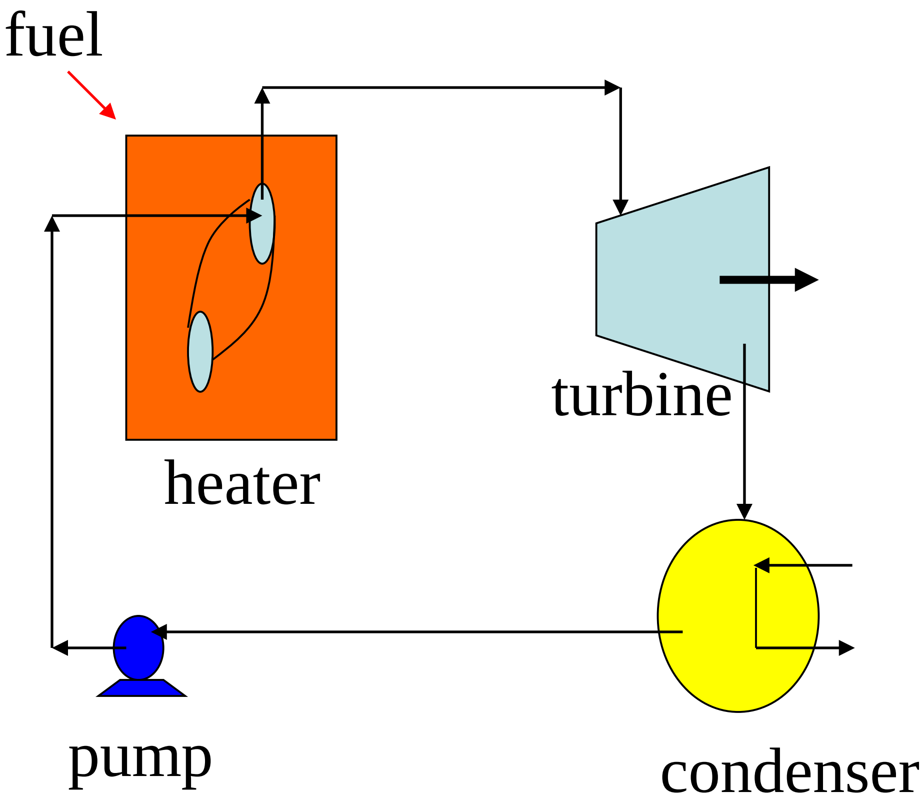
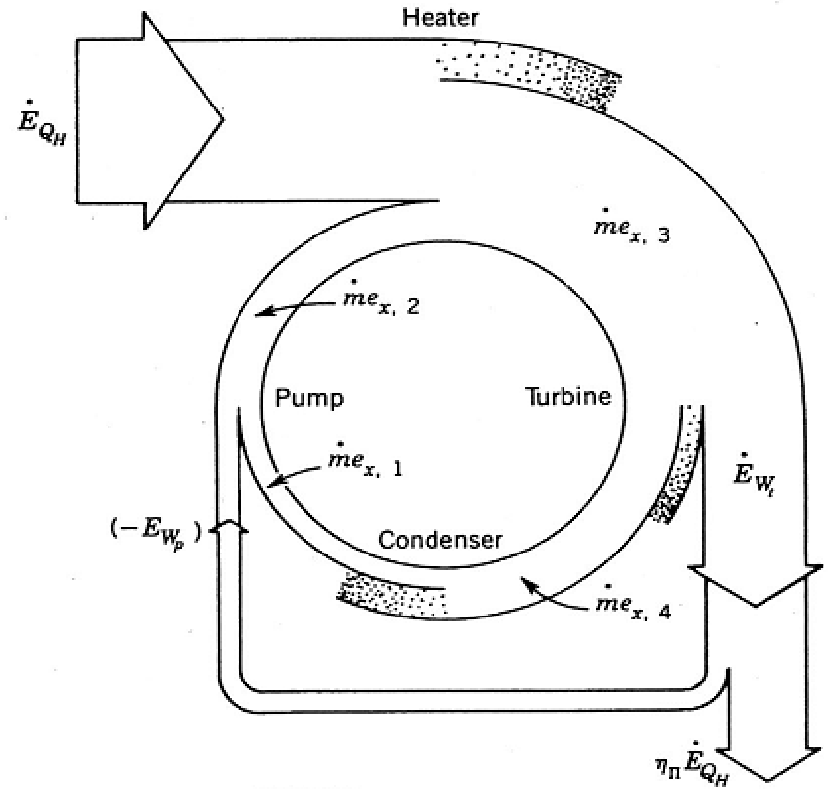
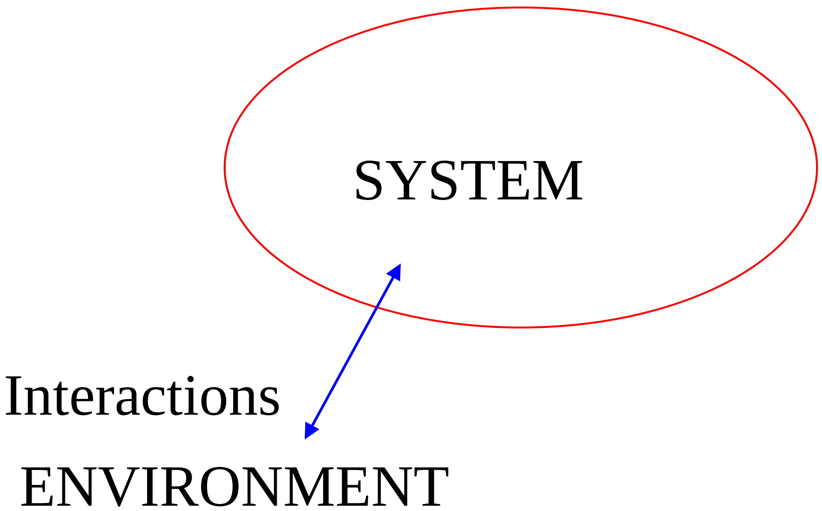




Each stream carries exergy (useful work content), which is the life blood of the power system. Exergy is destroyed (or entropy is generated) whenever streams interact with each other and with components. Our objective is to optimize the streams and components, so that they generate minimum entropy subject to the constraints.

THE METHOD OF THERMODYNAMIC OPTIMIZATION

Starts with:



-Thermodynamics provides the basic equations

-Flows, flow resistances, losses (irreversibility, “dissipation”) and interactions are integrated from related disciplines

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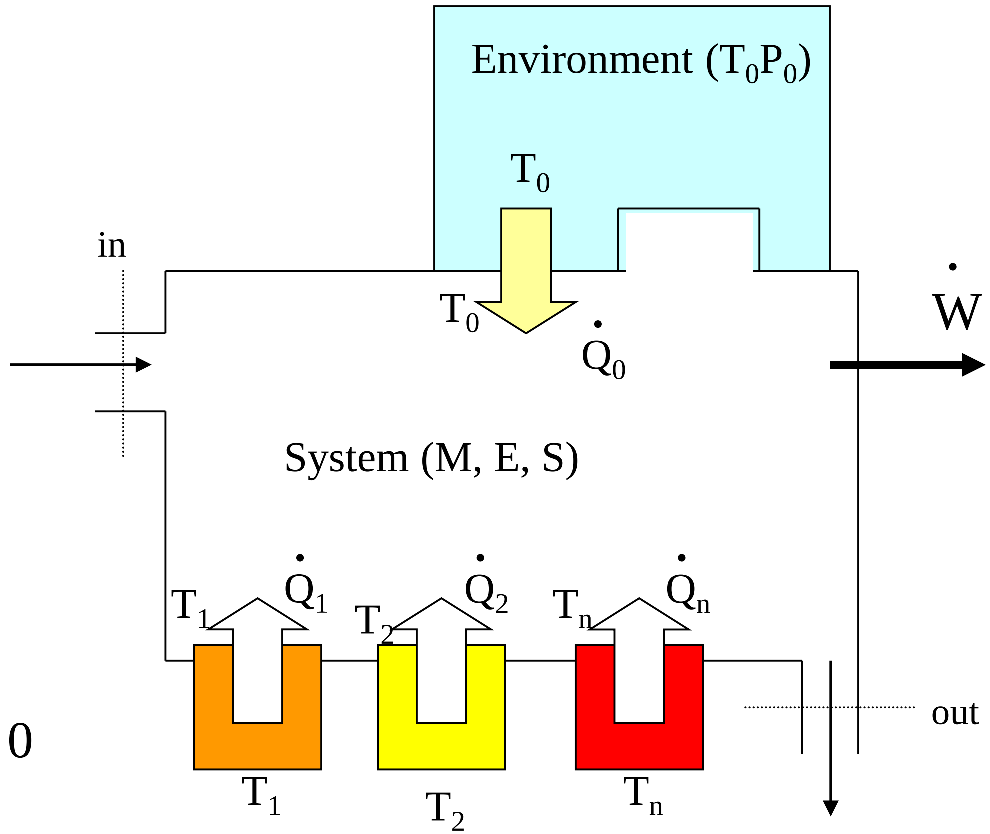
We start from the 1st and 2nd laws of thermodynamics:



$$\frac{dE}{dt} = \sum_{i=0}^{n} \dot{Q}_i - \dot{W} + \sum_{in} \dot{m}h^0 - \sum_{out} \dot{m}h^0$$

$$\dot{S}_{gen} = \frac{dS}{dt} - \sum_{i=0}^{n} \frac{\dot{Q}_i}{T_i} - \sum_{in} \dot{m}s + \sum_{out} \dot{m}s \geq 0$$

The two laws combined (eliminating $\dot{Q}_0$): *``Exergy Analysis''*

$$\dot{W} = -\frac{d}{dt}(E - T_0 S) + \sum_{i=1}^{n}\left(1 - \frac{T_0}{T_i}\right)\dot{Q}_i + \sum_{in}\dot{m}\left(h^0 - T_0 s\right) - \sum_{out}\dot{m}\left(h^0 - T_0 s\right) - T_0\dot{S}_{gen} \quad \text{(E1)}$$

destroyed exergy

Flow-exergy associated to mass flows

Actual work

Exergy, heat transfer interactions

Non-flow exergy

In the reversible limit ($\dot{S}_{gen} = 0$),

$$\dot{W}_{rev} = -\frac{d}{dt}(E - T_0 S) + \sum_{i=1}^{n}\left(1 - \frac{T_0}{T_i}\right)\dot{Q}_i + \sum_{in}\dot{m}\left(h^0 - T_0 s\right) - \sum_{out}\dot{m}\left(h^0 - T_0 s\right)$$

Work in the reversible limit

$$\dot{W} < \dot{W}_{rev}$$

Sustracting them we get the **Gouy-Stodola theorem**:
The destroyed power is proportional to the rate of entropy generation.

$$\dot{W}_{lost} = \dot{W}_{rev} - \dot{W} = T_0 \dot{S}_{gen} \qquad \text{(E.3)}$$

EGM starts from (E.3). We want to be as close as possible to the reversible limit ($\dot{W}_{rev}$), then we should work in the minimization of the entropy generation ($\dot{S}_{gen}$).

CONSTRAINED OPTIMIZATION

function, constraints and degrees of freedom

The FUNCTION to be optimized, is related to PURPOSE e.g:

-Max. power extraction
-Min. power requirement
-Max. of exergy collection
-Min. ratio destroyed exergy/ supplied exergy

CONSTRAINTS. e.g:
Total volume, area, material amount, operation temperatures.

DEGREES OF FREEDOM
-Operation temperatures
-Charging/discharging times
-Dimensions, thickness
-Spacing among components
-Material properties

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J.C. Ordóñez

The total surface constraint (c1), can be written as,

$$N = \mu N_s + \frac{U_s}{U_b} N_b + \mu' \frac{U_s}{U_w} N_w \qquad N = \frac{U_s A}{\dot{m} c_p} \text{ constant}$$

In the numerical computations, we defined the following area fractions

$$x = \frac{A_s}{A} \qquad \text{superheater (steam)}$$

$$y = \frac{A_b}{A} \qquad \text{boiling}$$

$$1 - x - y = \frac{A_w}{A} \qquad \text{Preheater (liq. water)}$$

The equations we have until now allow us to compute the temperature distribution. We need the work output.

Constrained optimization:

The total area constraint, can be written as,

$$N = \mu N_s + \frac{U_s}{U_b} N_b + \mu' \frac{U_s}{U_w} N_w \qquad N = \frac{U_s A}{\dot{m} c_p} \text{ constant}$$

In the numerical computations, we defined the following area fractions

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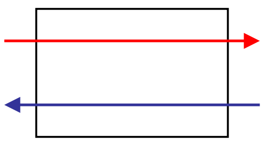
The equations we have until now allow us to compute the temperature distribution. We need the work output.

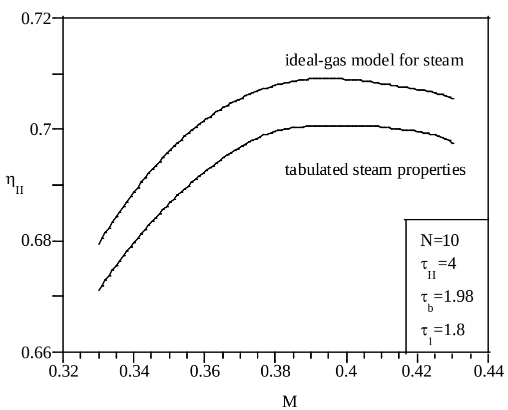
Concluding remarks (1/3):

• The **extraction of power** from a hot stream **can be maximized** by properly **matching** the stream with a receiving stream of cold fluid, across a finite-size heat transfer area

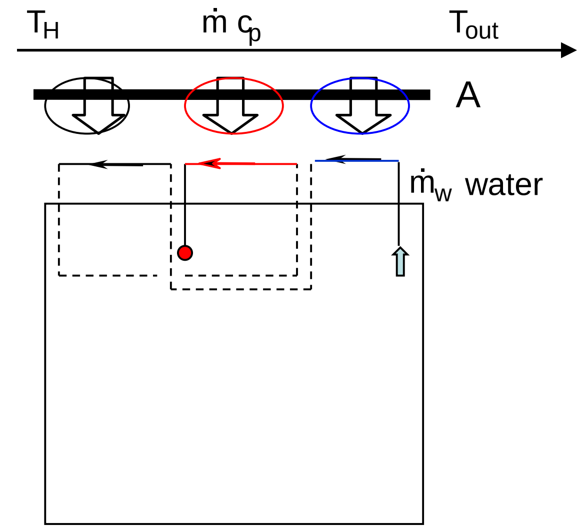


counterflow

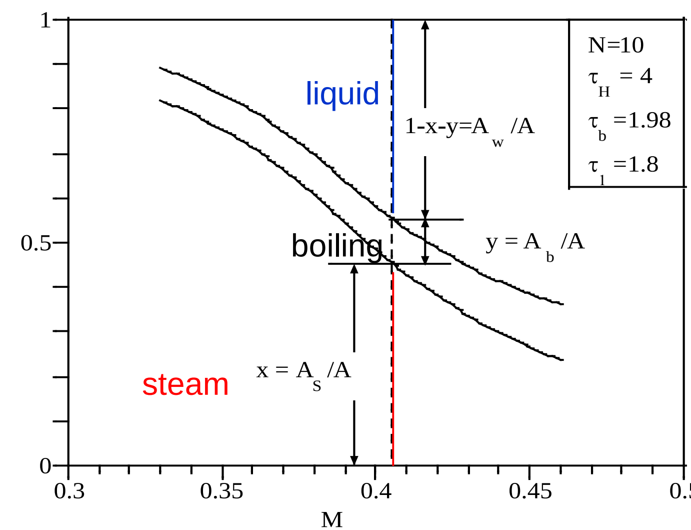
+



Optimal mass flow rate ratio



There is an associated optimal allocation of heat exchanger inventory:



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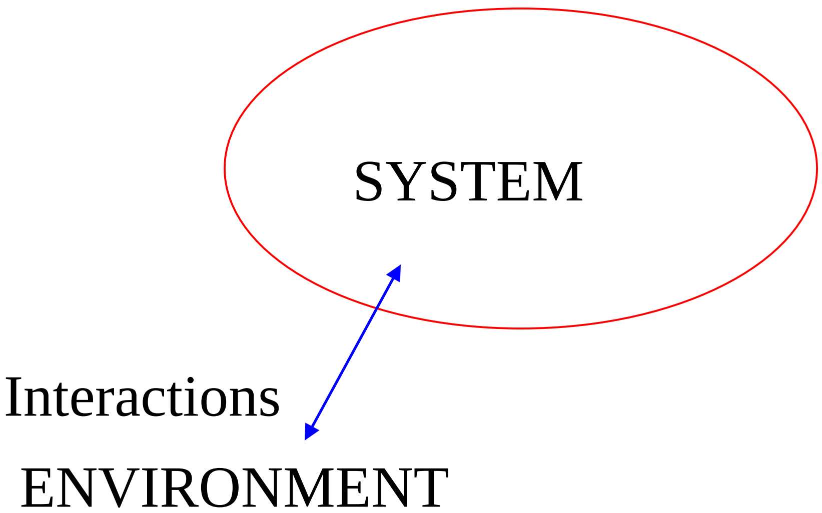
We want to search for an optimal matching between the streams. → Thermodynamic optimization

Starts with:



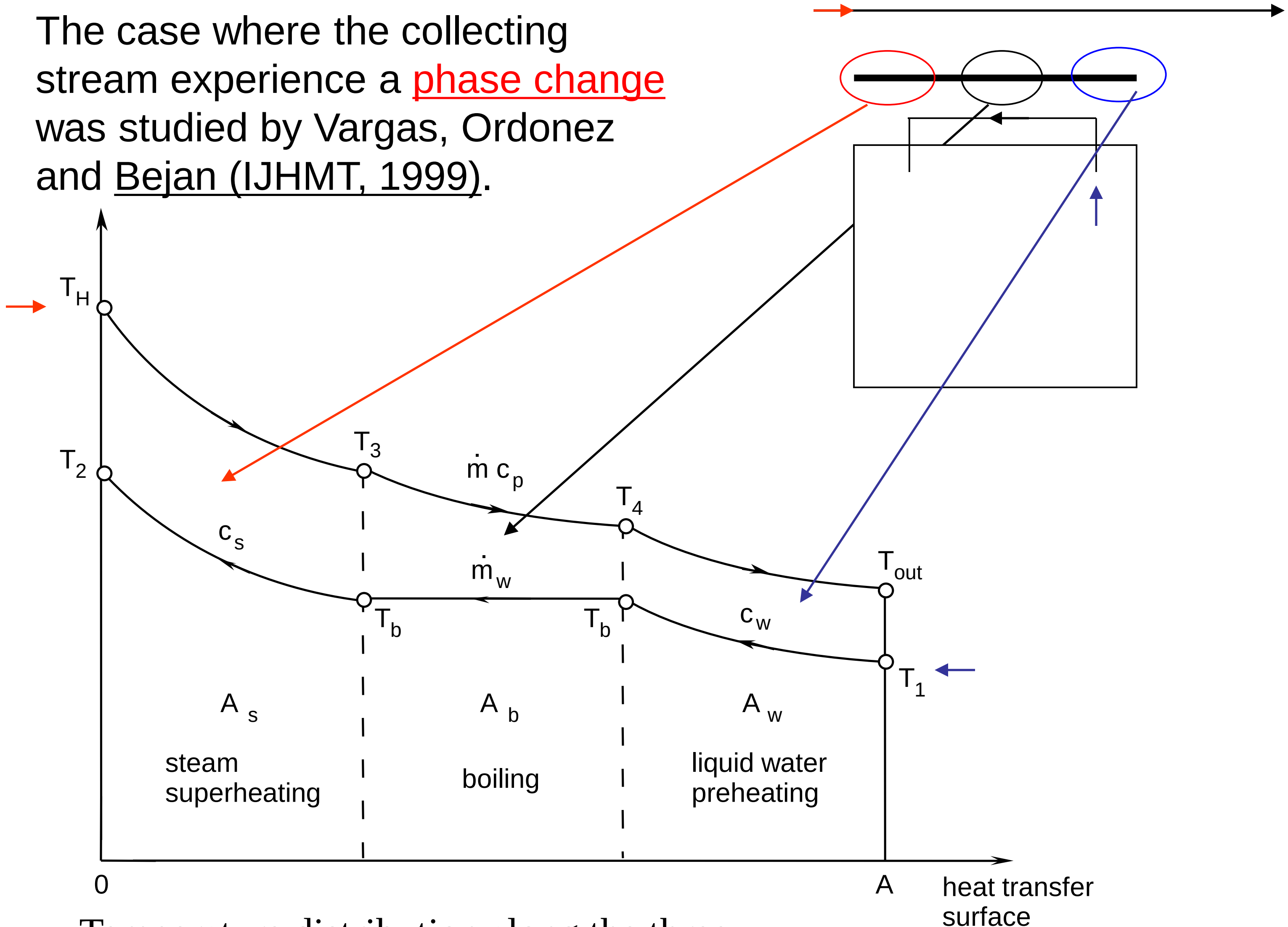
-Thermodynamics provides the basic equations

-Flows, flow resistances, losses (irreversibility, “dissipation”) and interactions are integrated from related disciplines

MODEL → OPTIMIZED SYSTEM

CONSTRAINED OPTIMIZATION

The case where the collecting stream experience a phase change was studied by Vargas, Ordonez and Bejan (IJHMT, 1999).



Temperature distribution along the three sections of the heat exchanger

Heat transfer analysis
classical effectiveness Ntu analysis

$$\mu < 1, \mu' < 1$$

Superheater

$$\varepsilon_s = \frac{1-\exp[-N_s(1-\mu)]}{1-\mu\exp[-N_s(1-\mu)]}$$

$$\varepsilon_s = \frac{T_H - T_3}{\mu(T_H - T_b)}$$

$$\varepsilon_s = \frac{T_2 - T_b}{T_3 - T_b}$$

$$\mu = \frac{\dot{m}_w c_s}{\dot{m} c_p}$$

$$N_s = \frac{U_s A_s}{\dot{m}_w c_s}$$

Boiling

$$\varepsilon_b = 1 - \exp(-N_b)$$

$$\varepsilon_b = \frac{T_H - T_3}{T_H - T_b}$$

$$T_H - T_3 = \mu \frac{h_{fg}}{c_s}$$

$$N_b = \frac{U_b A_b}{\dot{m} c_p}$$

Preheating

$$\varepsilon_w = \frac{1-\exp[-N_w(1-\mu')]}{1-\mu\exp[-N_w(1-\mu')]}$$

$$\varepsilon_w = \frac{T_b - T_1}{T_4 - T_1}$$

$$\varepsilon_w = \frac{T_{out} - T_4}{\mu'(T_1 - T_4)}$$

$$\mu' = \frac{\dot{m}_w c_w}{\dot{m} c_p}$$

$$N_w = \frac{U_w A_w}{\dot{m}_w c_w}$$

MODEL ⇨ OPTIMIZED SYSTEM

CONSTRAINED OPTIMIZATION

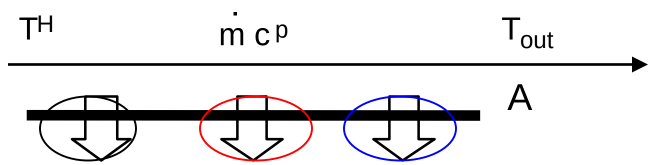
Constrained optimization:

<u>Maximize</u> power extraction (efficiency)

For <u>fixed</u> total area, A

<u>Degree of freedom</u>, $\dot{m}_w$

The total area constraint



can be written as,

$$N = \mu N_s + \frac{U_s}{U_b} N_b + \mu' \frac{U_s}{U_w} N_w$$

$$N = \frac{U_s A}{\dot{m} c_p}$$

In the numerical computations, we defined the following area fractions

$$x = \frac{A_s}{A}$$ superheater (steam)

$$y = \frac{A_b}{A}$$ boiling

$$1 - x - y = \frac{A_w}{A}$$ Preheater (liq. water)

MODEL

CONSTRAINED OPTIMIZATION

OPTIMIZED SYSTEM

Heat transfer analysis
classical effectiveness Ntu analysis

$$\mu < 1, \mu' < 1$$

Superheater

$$\varepsilon_s = \frac{1-\exp[-N_s(1-\mu)]}{1-\mu\exp[-N_s(1-\mu)]}$$

$$\varepsilon_s = \frac{T_H - T_3}{\mu(T_H - T_b)}$$

$$\varepsilon_s = \frac{T_2 - T_b}{\mu(T_H - T_b)}$$

$$\mu = \frac{\dot{m}_w c_s}{\dot{m}c_p}$$

$$N_s = \frac{U_s A_s}{\dot{m}_w c_s}$$

Boiling

$$\varepsilon_b = 1-\exp(-N_b)$$

$$\varepsilon_b = \frac{T_4 - T_3}{T_3 - T_b}$$

$$\dot{m}h_{fg} = \dot{m}c_p(T_3 - T_b)$$

$$N_b = \frac{U_b A_b}{\dot{m}c_p}$$

Preheating

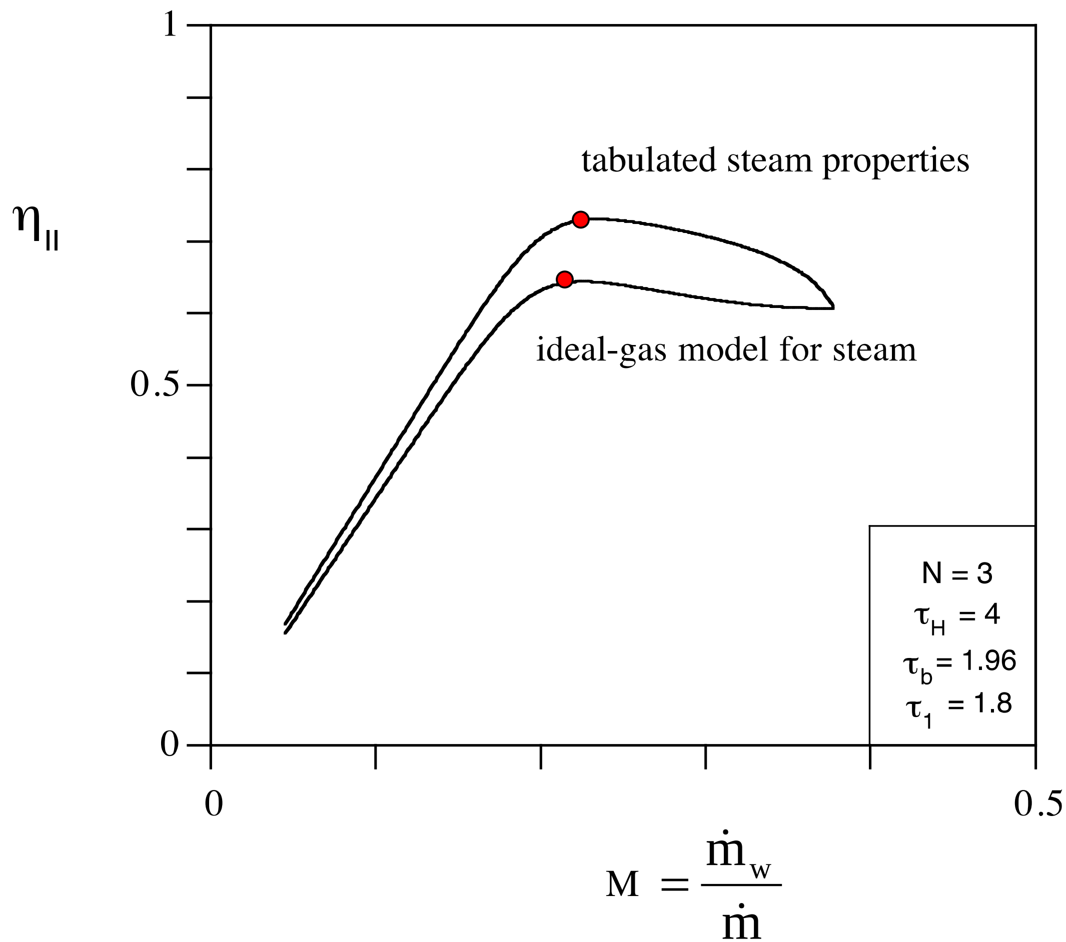
$$\varepsilon_w = \frac{1-\exp[-N_w(1-\mu')]}{1-\mu\exp[-N_w(1-\mu')]}$$

$$\varepsilon_w = \frac{T_b - T_1}{T_4 - T_1}$$

$$\varepsilon_w = \frac{T_{out} - T_4}{\mu'(T_1 - T_4)}$$

$$\mu = \frac{\dot{m}_w cw}{\dot{m}c_p}$$

$$N_w = \frac{U_w A_w}{\dot{m}_w c_w}$$



Maximization of the second law efficiency by selecting the mass flow rate of the water stream

Duke University
Mechanical Engineering and Material Science Department
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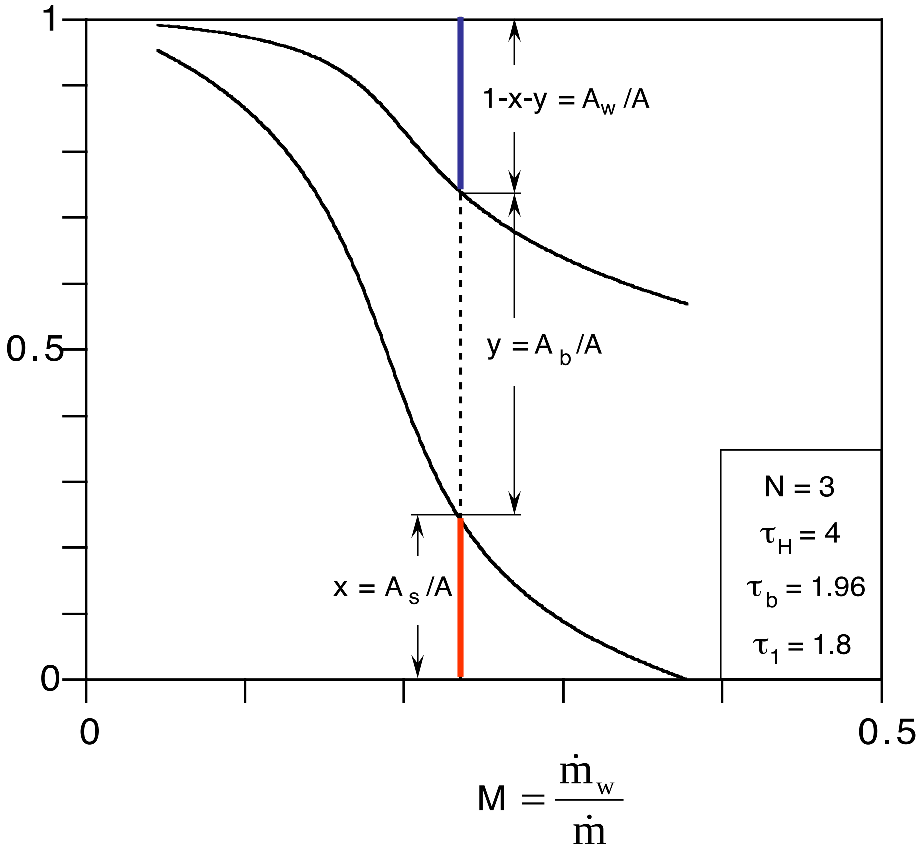
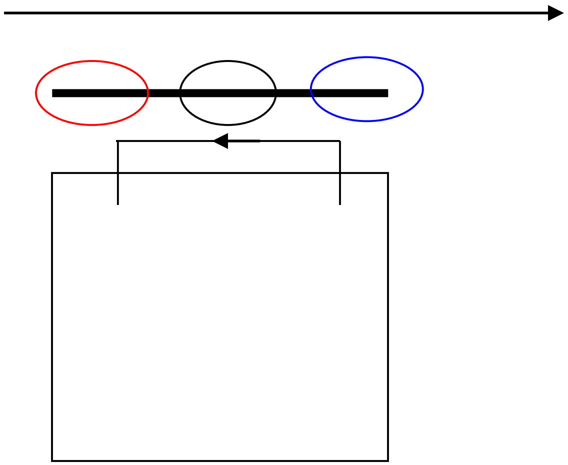
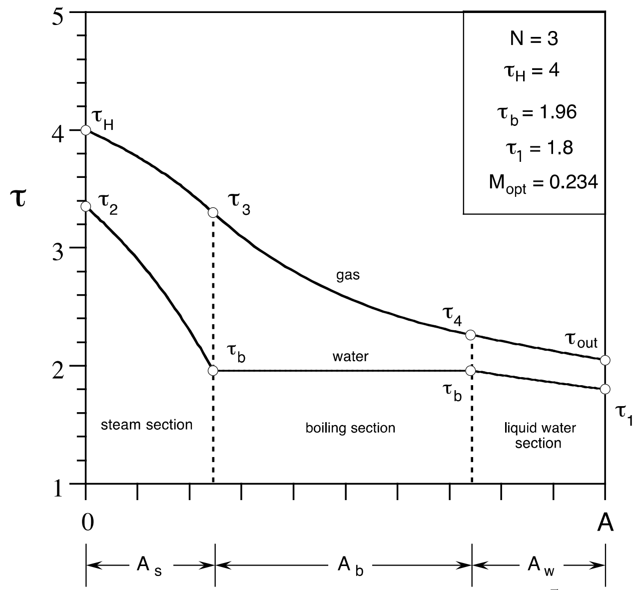
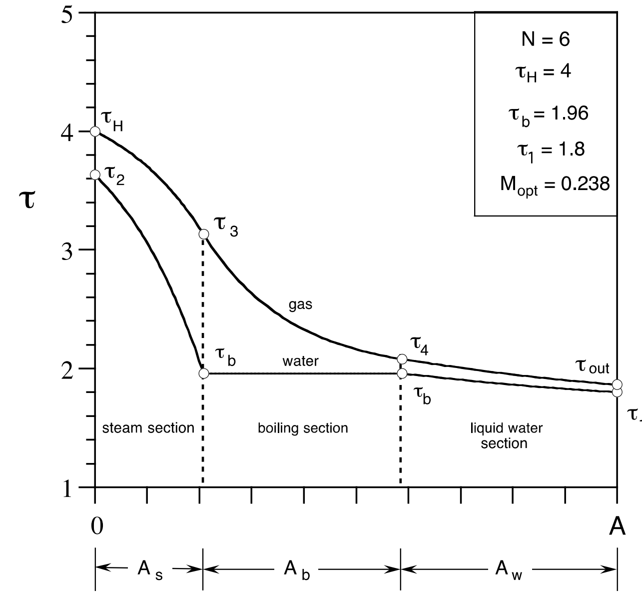




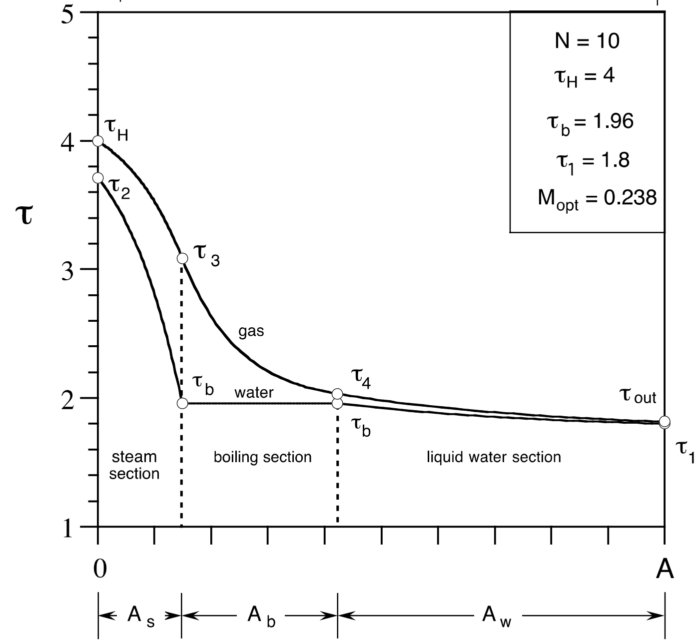
Effect of the mass flow rate on the allocation of area among the sections of the heat exchanger



(a)



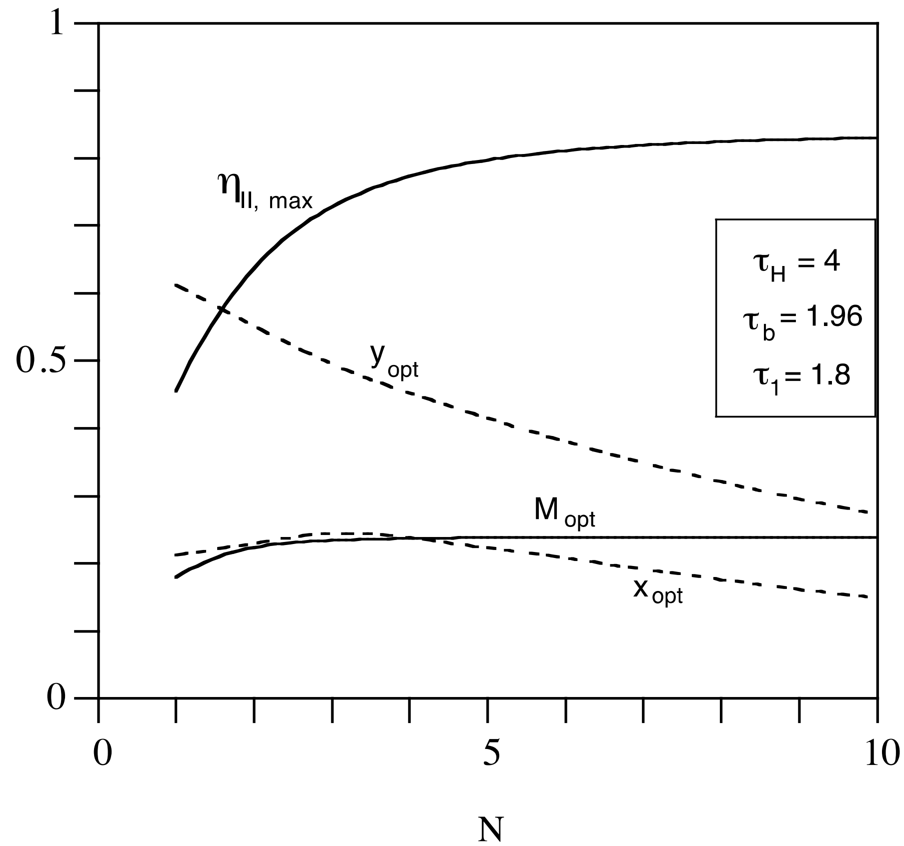
(b)



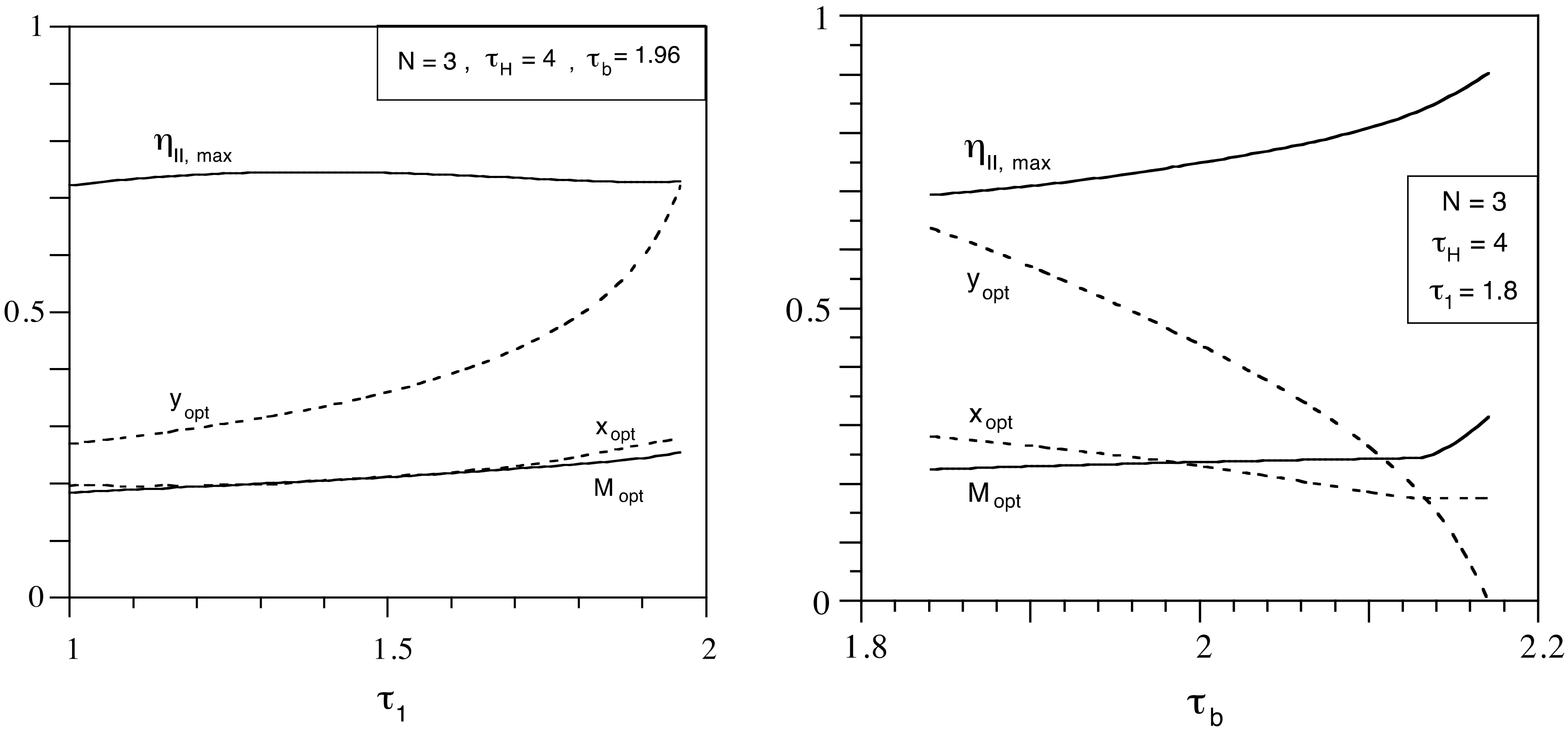
(c)

Effect of the heat transfer area size on the "match" between the temperature distributions of the two streams

J.C. Ordóñez



Effect of heat exchanger size on the second law efficiency and on the allocation of heat transfer area



Effects of varying the working-fluid inlet temperature and boiling temperature

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Mechanical Engineering and Material Science Department
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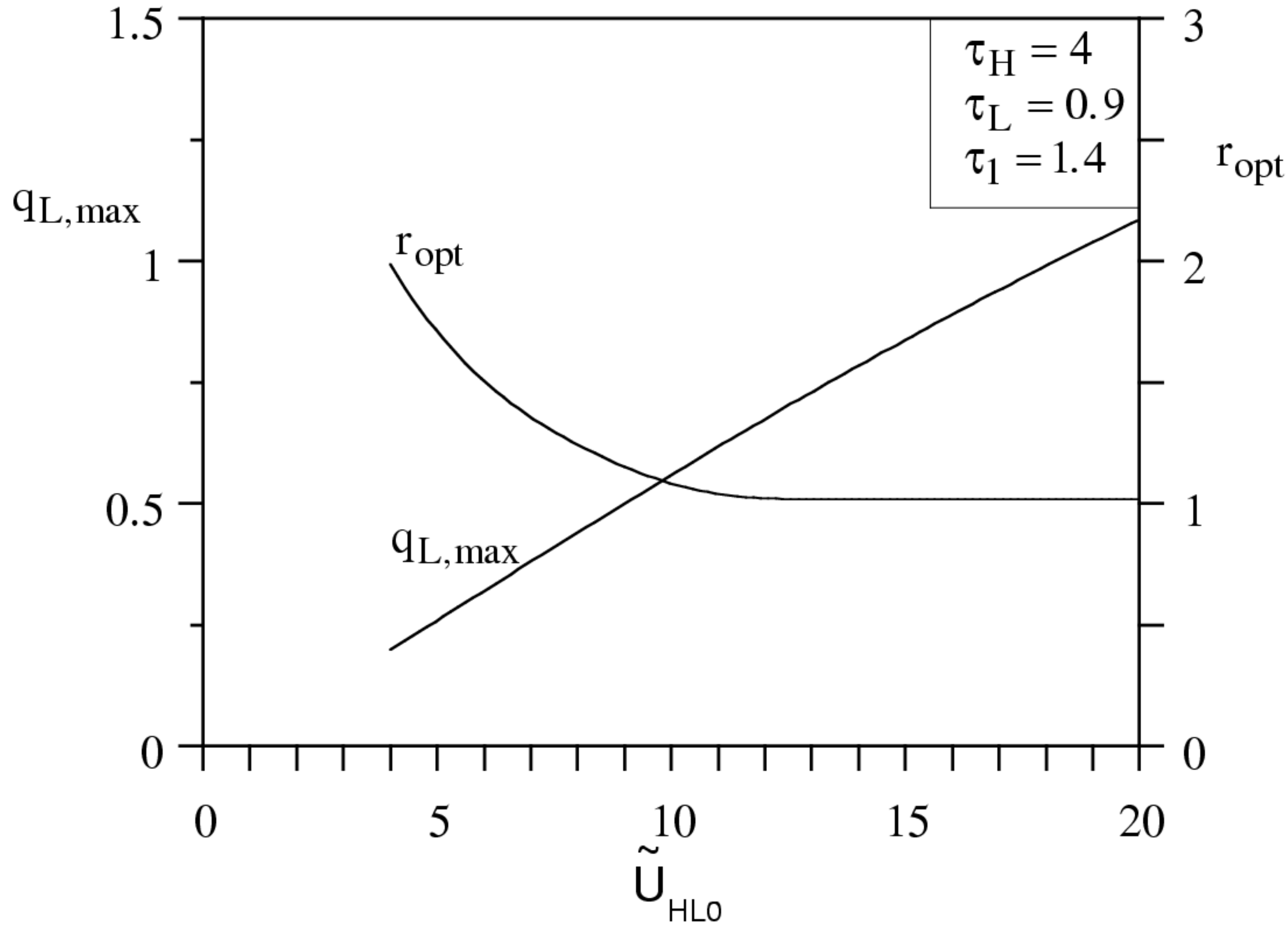
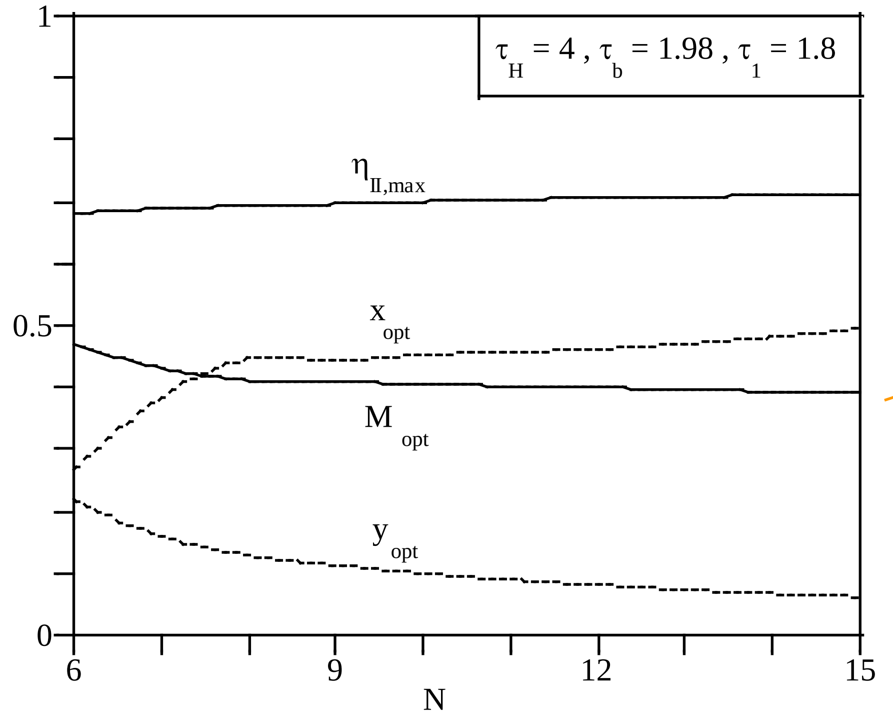
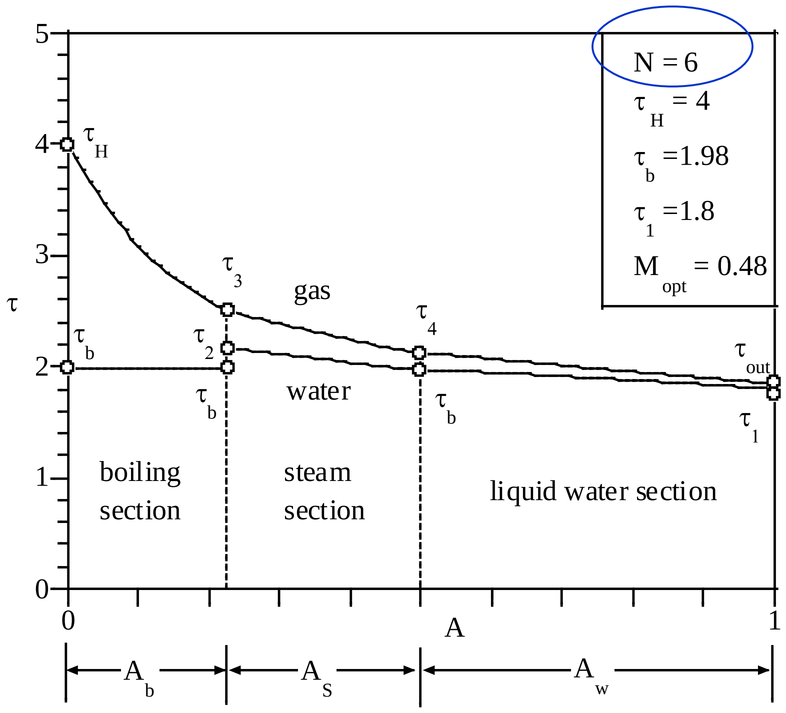


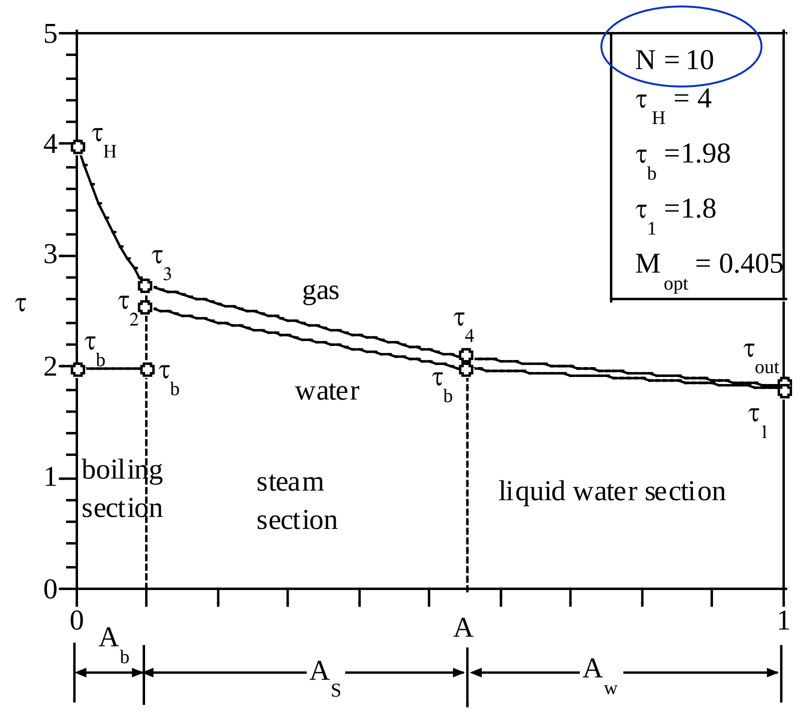
1.5
1
0.5
0
3
2
1
0
$q_{L,max}$
r_{opt}
r_{opt}
$q_{L,max}$
$\tau_H = 4$
$\tau_L = 0.9$
$\tau_1 = 1.4$
0
5
10
15
20
$\tilde{U}_{HLo}$



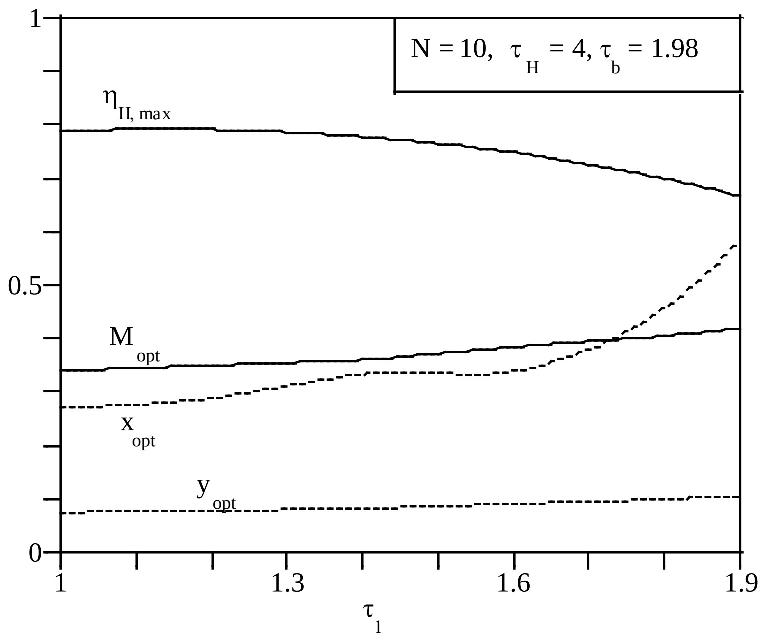
Notice "robustness" of the optimal 'match'

Effect of heat exchanger size on the second law efficiency and on the allocation of heat transfer area

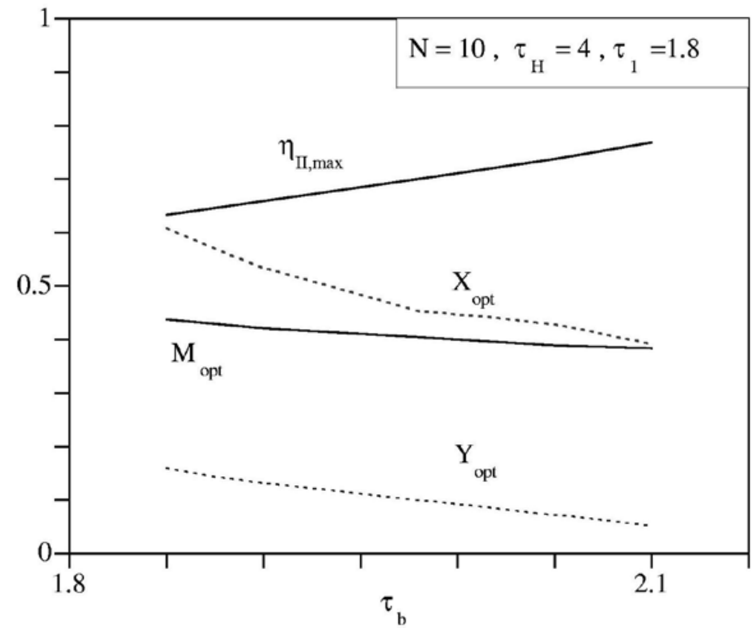




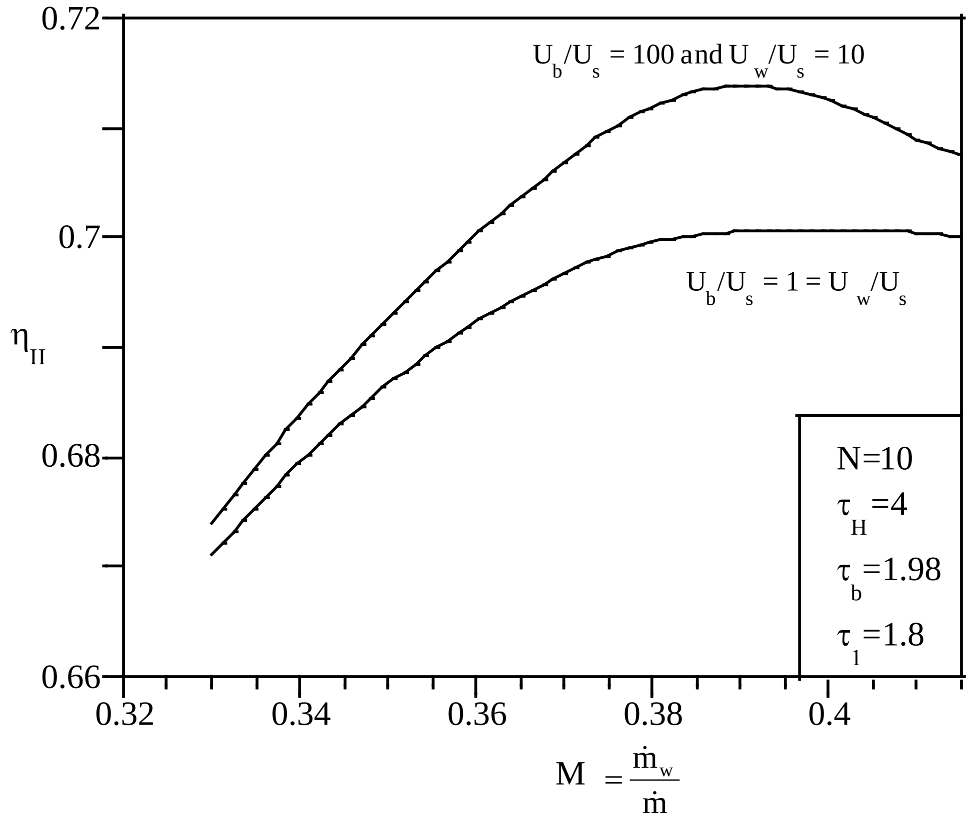
Effect of the heat transfer area size on the "match" between the temperature distributions of the two streams



Effect of varying the working-fluid inlet temperature



Effect of varying the boiling temperature



Effect of overall heat transfer coefficients on the second law efficiency

Concluding remarks :

Optimal matching among the hot and collecting stream

(counterflow configuration, optimal mass flow rate ratio, optimal allocation of heat exchanger inventory)

Collecting stream is single phase

IJHMT Bejan and Errera, 1998

Collecting stream experience phase change

IJHMT Vargas, Ordonez and Bejan, 1999

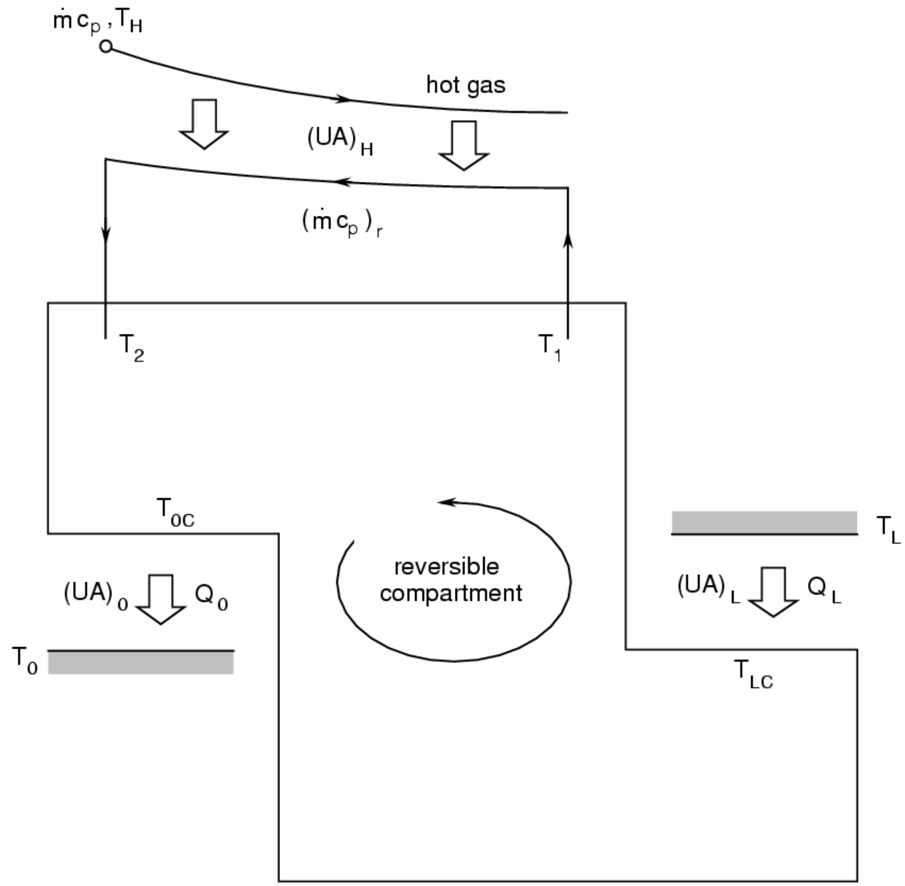
Collecting stream experience phase change and boiling section is in contact with hottest gases

ASME HT Charlotte Ordonez and Chen, 2004

TABLE I. Comparison among observed efficiencies η_{exp}, with the theoretical η_C and Curzon-Ahlborn, η_{CA}, values. Data taken from [20].

Power Plant	T_c(K)	T_h(K)	τ	η_{exp}	η_C	η_{CA}
Doel 4 (Nuclear, Belgium)	566	283	0.50	0.35	0.50	0.31
Almaraz II (Nuclear, Spain)	600	290	0.48	0.34	0.52	0.31
Sizewell B (Nuclear, UK)	581	288	0.50	0.36	0.50	0.30
Cofrentes (Nuclear, Spain)	562	289	0.51	0.34	0.49	0.29
Heysham (Nuclear, UK)	727	288	0.40	0.40	0.60	0.37
West Thurrock (Coal, UK)	838	298	0.36	0.36	0.64	0.40
CANDU (Nuclear, Canada)	573	298	0.52	0.30	0.48	0.28
Larderello (Geothermal, Italy)	523	353	0.68	0.16	0.32	0.18
Calder Hall (Nuclear, UK)	583	298	0.51	0.19	0.49	0.29
(Steam/Mercury, USA)	783	298	0.38	0.34	0.62	0.38
(Steam, UK)	698	298	0.43	0.28	0.57	0.35
(Gas Turbine, Switzerland)	963	298	0.31	0.32	0.69	0.44
(Gas Turbine, France)	953	298	0.31	0.34	0.69	0.44

3. Optimal Matching for Refrigeration:

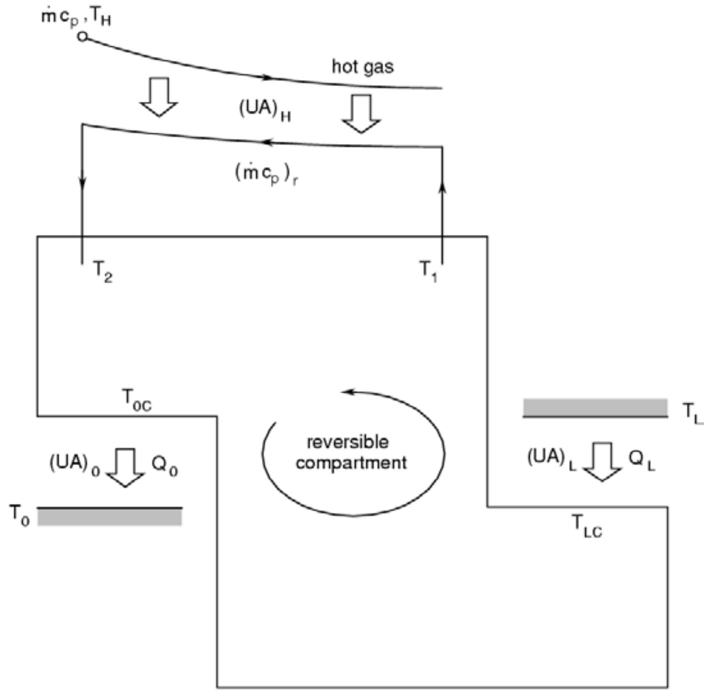


Refrigeration system driven by a hot stream through a counterflow heat exchanger

Thermodynamics

$$\left(\dot{m}c_p\right)_r\left(T_2 - T_1\right) + \dot{Q}_L - \dot{Q}_0 = 0$$

$$\left(\dot{m}c_p\right)_r \ln\frac{T_2}{T_1} + \frac{\dot{Q}_L}{T_{LC}} - \frac{\dot{Q}_0}{T_{0C}} = 0$$



Constraint:

$$A = A_H + A_0 + A_L$$

Heat Transfer:

$$\dot{Q}_0 = (UA)_0(T_{0C} - T_0)$$

$$\dot{Q}_L = (UA)_L(T_L - T_{LC})$$

$$N_H = \frac{U_H A_H}{\dot{m}c_p} \qquad r = \frac{\dot{m}c_p}{\left(\dot{m}c_p\right)_r}$$

r <1

$$\varepsilon = \frac{1 - \exp\left[-N_H\left(1 - r\right)\right]}{1 - r\exp\left[-N_H\left(1 - r\right)\right]}$$

$$T_2 - T_1 = \varepsilon r\left(T_H - T_1\right)$$

r >1

$$\varepsilon = \frac{1 - \exp\left[-N_H\left(1 - r^{-1}\right)\right]}{1 - r^{-1}\exp\left[-N_H\left(1 - r^{-1}\right)\right]}$$

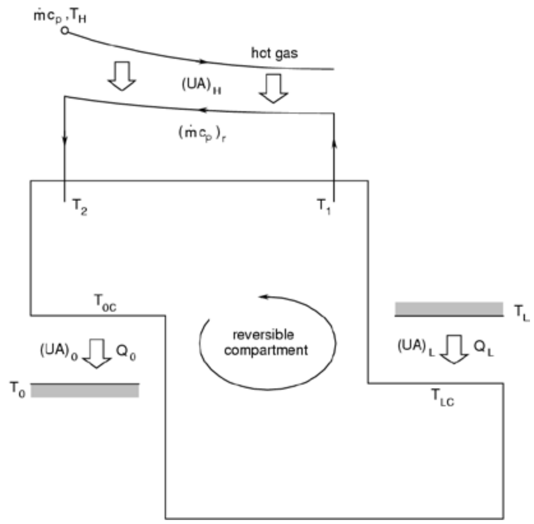
$$T_2 - T_1 = \varepsilon\left(T_H - T_1\right)$$

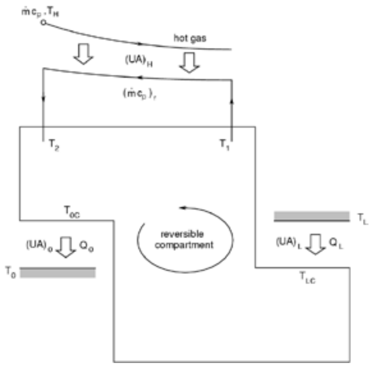
Dimensionless groups:

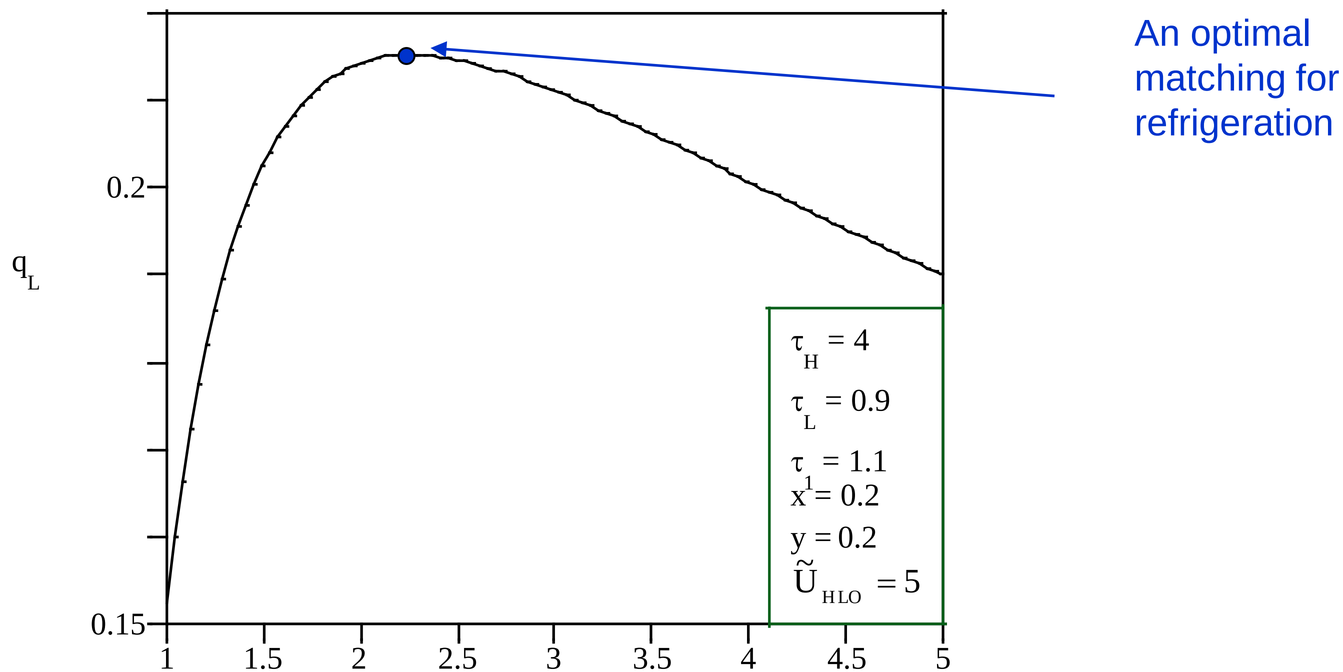
$$\tau = \frac{T}{T_0} \qquad q_0 = \frac{\dot{Q}_0}{\dot{m} c_p T_0} \qquad q_L = \frac{\dot{Q}_L}{\dot{m} c_p T_0}$$

$$\tilde{U}_0 = \frac{U_0 A}{\dot{m} c_p} \qquad \tilde{U}_H = \frac{U_H A}{\dot{m} c_p} \qquad \tilde{U}_L = \frac{U_L A}{\dot{m} c_p}$$

$$x = \frac{A_H}{A} \qquad y = \frac{A_L}{A} \qquad 1 - x - y = \frac{A_0}{A}$$

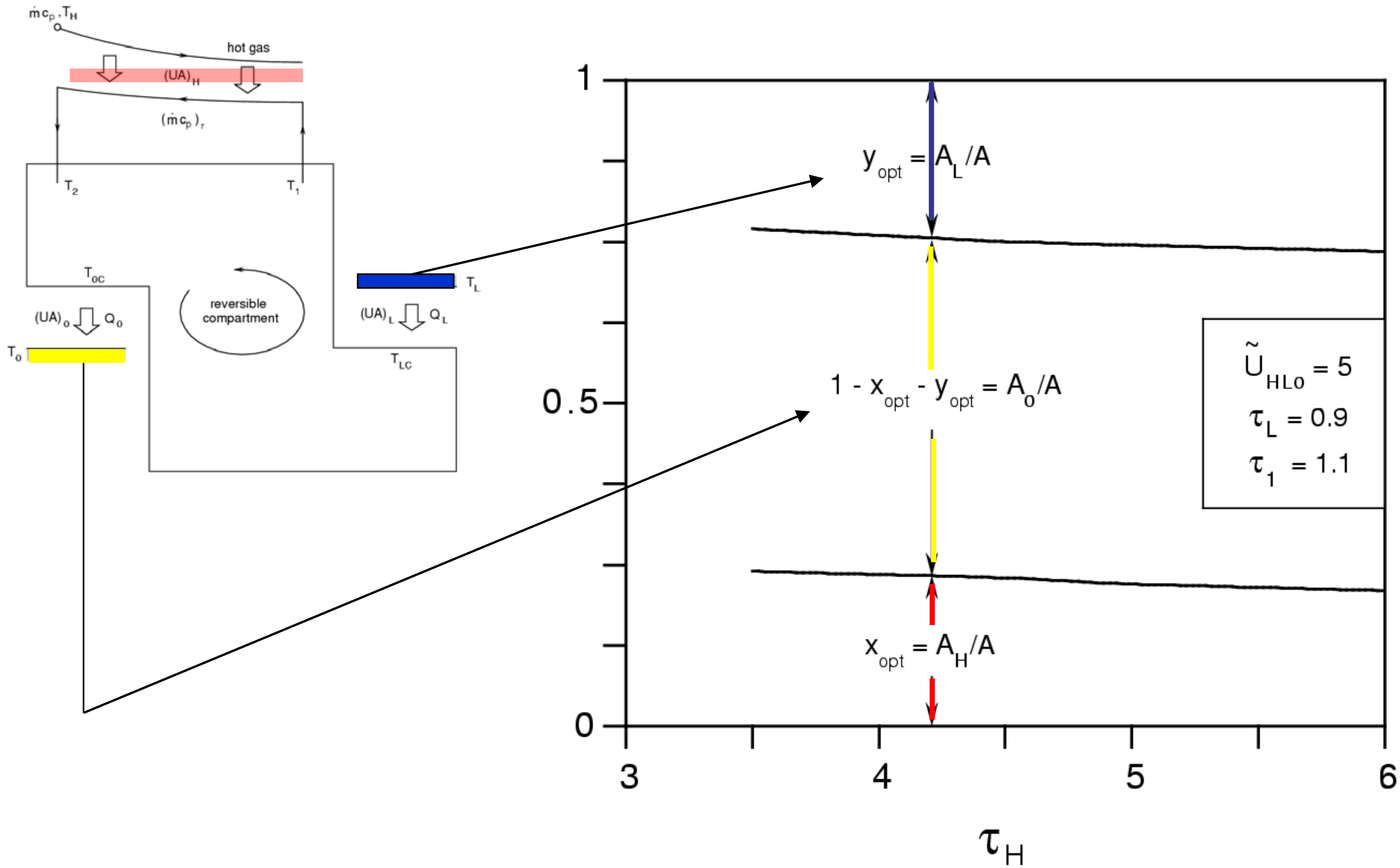




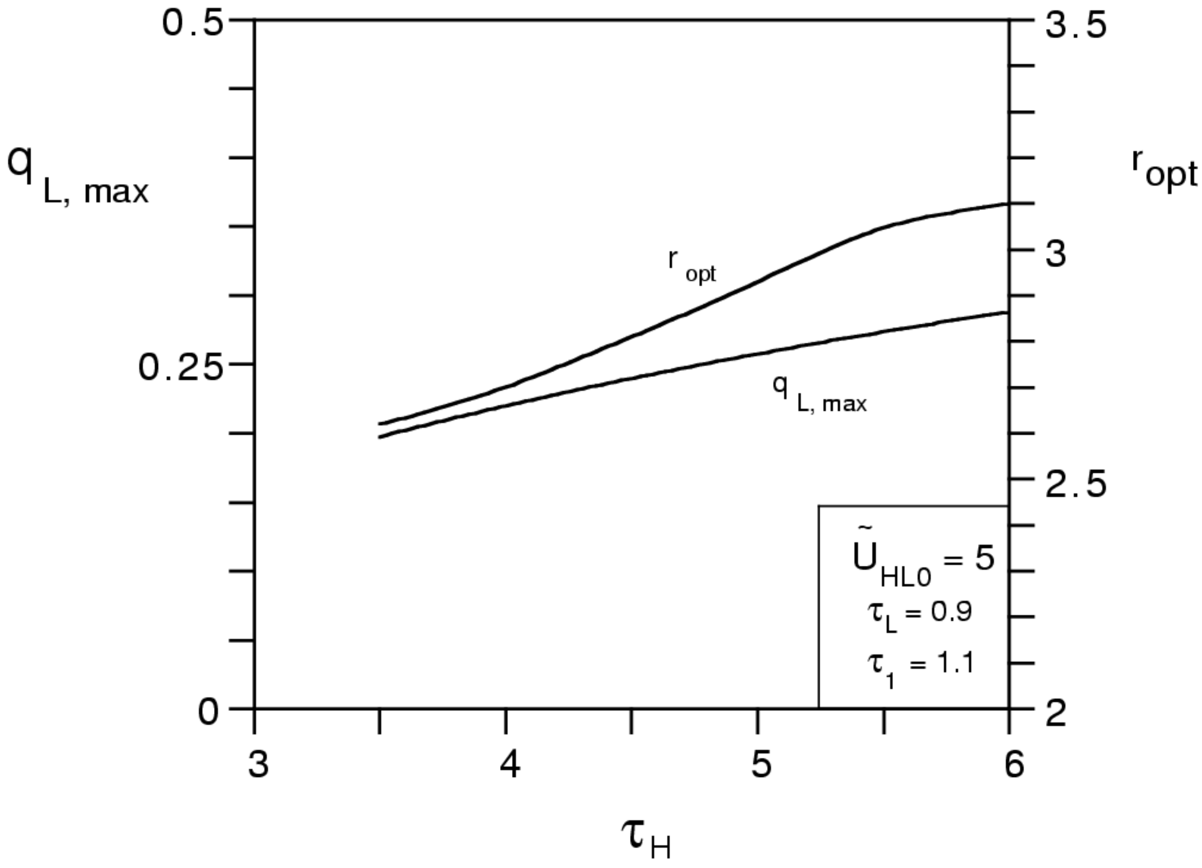


$$r = \frac{\dot{m}c_p}{(\dot{m}c_p)_r}$$

Illustration of the existence of an optimal capacity rate ratio

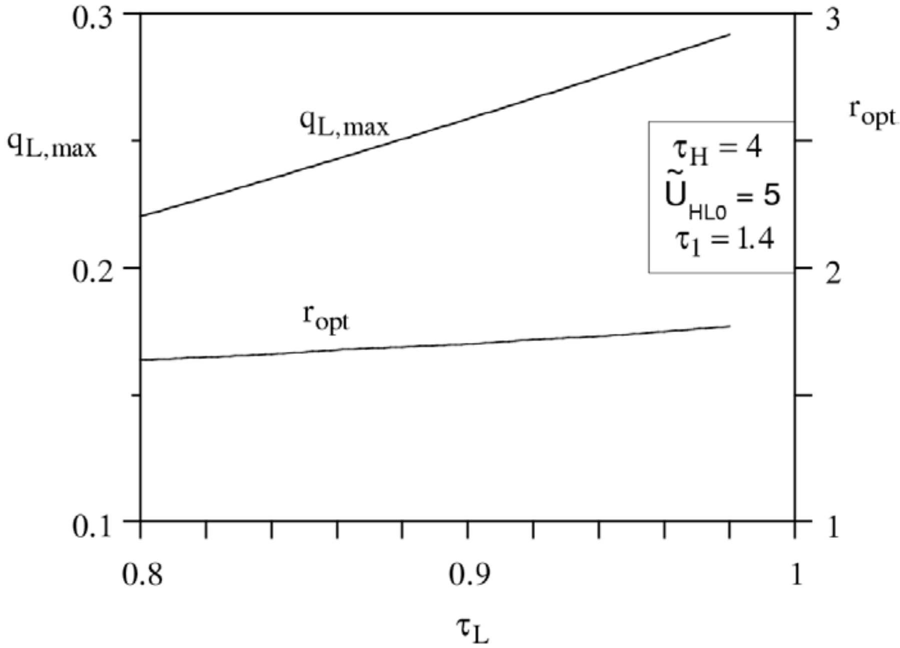


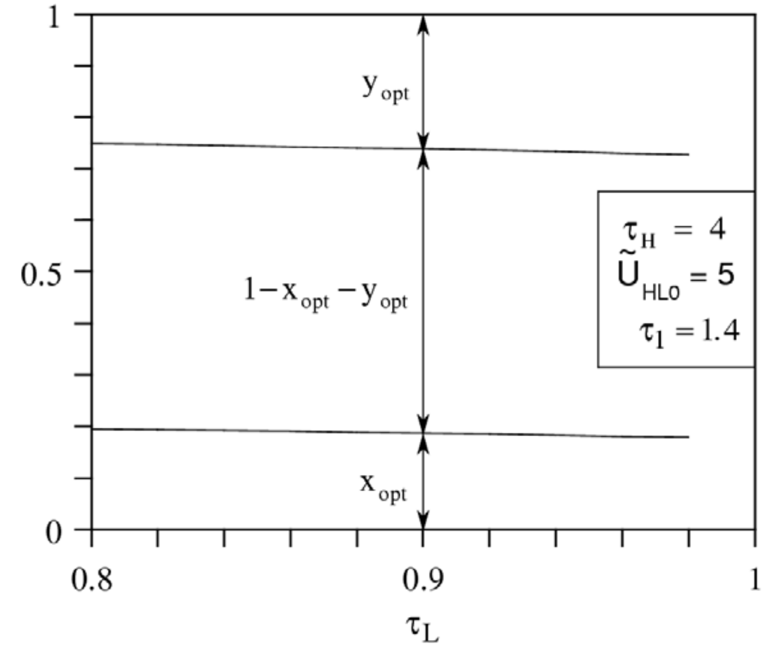
THE EFFECT OF THE HOT-STREAM INLET TEMPERATURE ON THE OPTIMAL ALLOCATION OF HEAT EXCHANGER INVENTORY



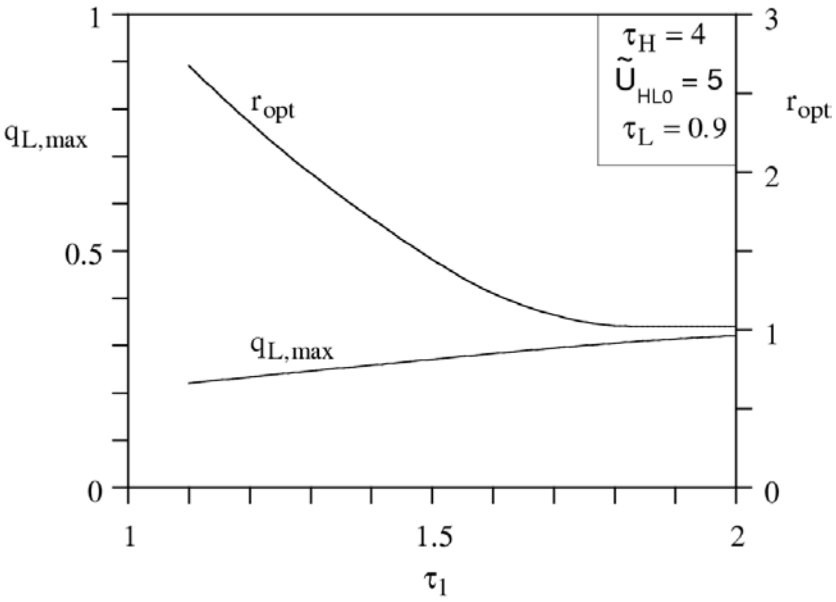
Here the heat exchanger area allocation has been optimized

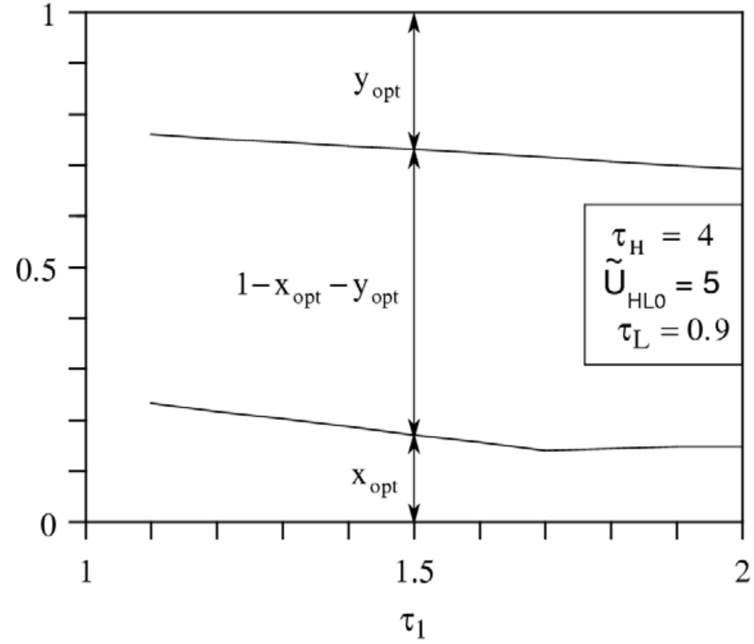
Maximized refrigeration rate and optimal capacity rate ratio of the countreflow system



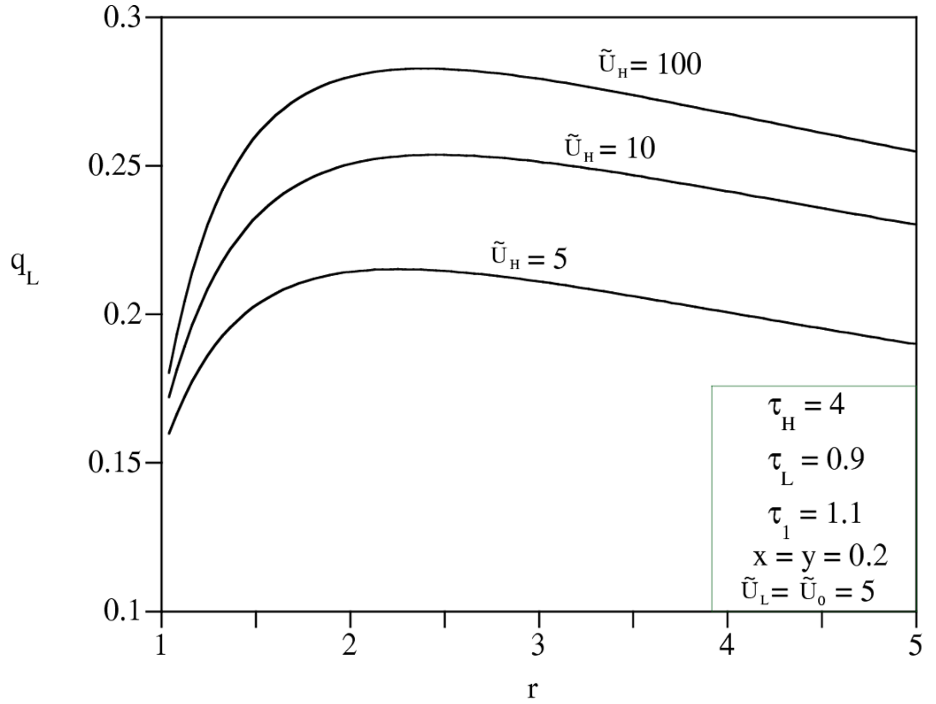


Effects of refrigeration temperature



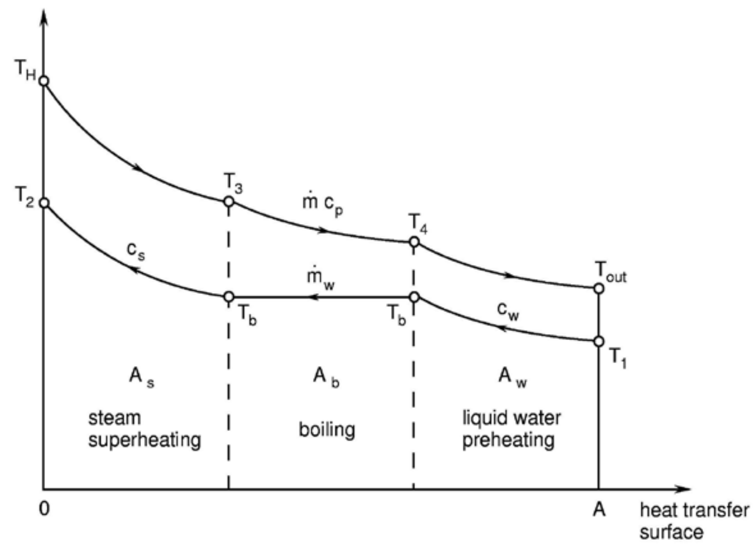


Effects of the matching stream inlet temperature



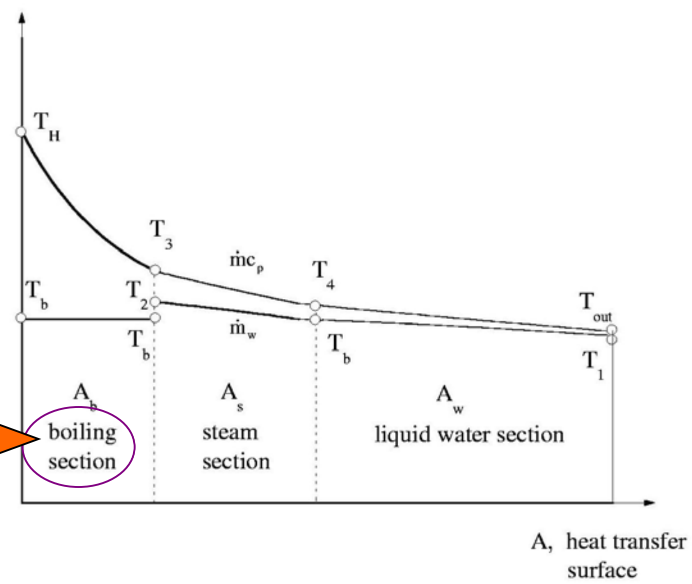
EFFECT OF HOT SIDE OVERALL HEAT TRANSFER COEFFICIENT ON THE REFRIGERATION RATE AND THE EXISTENCE OF AN OPTIMAL CAPACITY RATE RATIO.

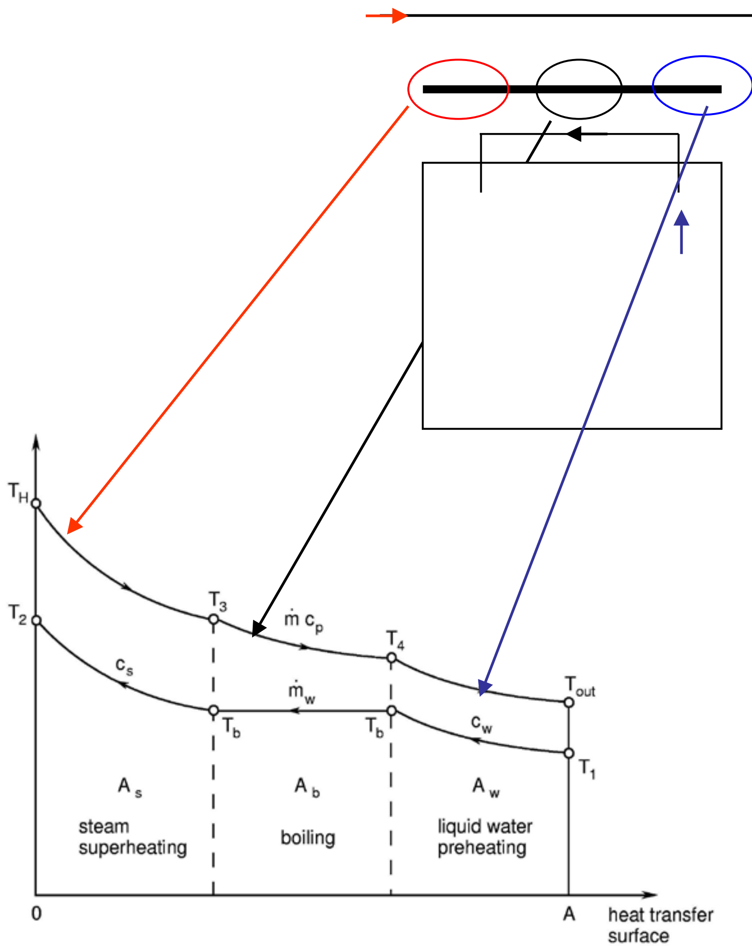
2. Phase change under limiting collecting temperatures

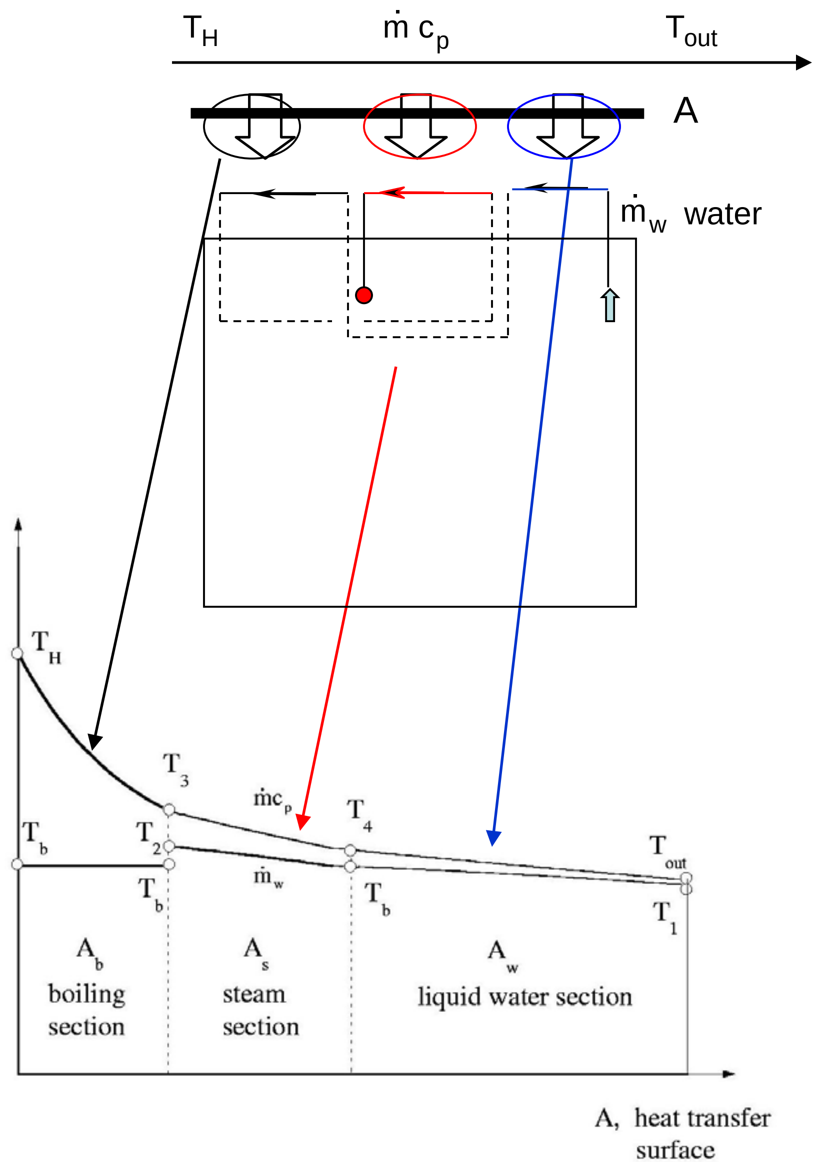


Placing the boiling section in contact with the hottest gases will prevent pipe overheating (materials constraints).

An alternative configuration:





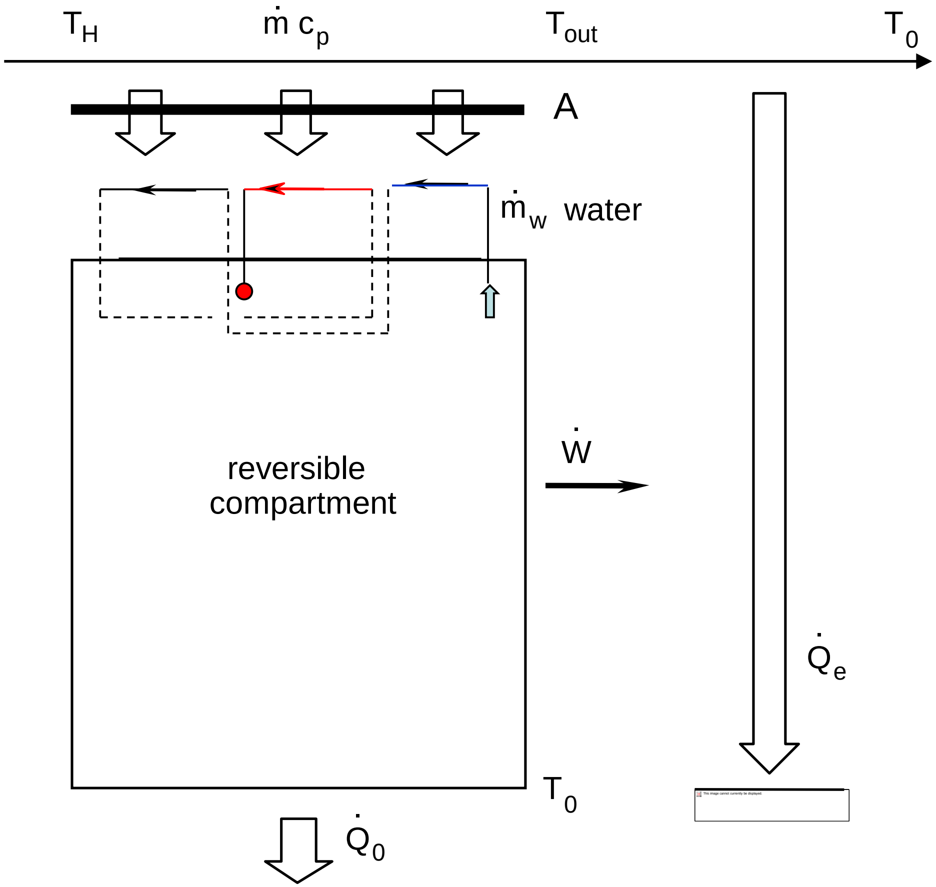


Temperature distribution along the three sections of the heat exchanger

MODEL → OPTIMIZED SYSTEM

CONSTRAINED OPTIMIZATION

Entropy generation analysis



$$\dot{W} = \dot{m}\left(h_H - h_0\right) - \dot{Q}_0 - \dot{Q}_e$$

$$\dot{S}_{gen} = \frac{\dot{Q}_0 + \dot{Q}_e}{T_0} + \dot{m}\left(s_0 - s_H\right) \geq 0$$

$$\dot{W} = \dot{m} e_{x,H} - T_0 \dot{S}_{gen}$$

$$e_{x,H} = \left(h_H - T_0 s_H\right) - \left(h_0 - T_0 s_0\right)$$

Now for the heat exchanger, and external cooling alone:

$$\dot{m}\left(h_H - h_0\right) - \dot{m}_w\left(h_2 - h_1\right) - \dot{Q}_e = 0$$

$$\dot{S}_{gen} = \dot{m}\left(s_0 - s_H\right) + \dot{m}_w\left(s_2 - s_1\right) + \frac{\dot{Q}_e}{T_0} \geq 0$$

$$T_0 \dot{S}_{gen} = \dot{m} e_{x,H} - \dot{m}_w\left(e_{x,2} - e_{x,1}\right)$$

$$\dot{W} = \dot{m}\left(e_{x,2} - e_{x,1}\right)$$

We can maximize the power output using:

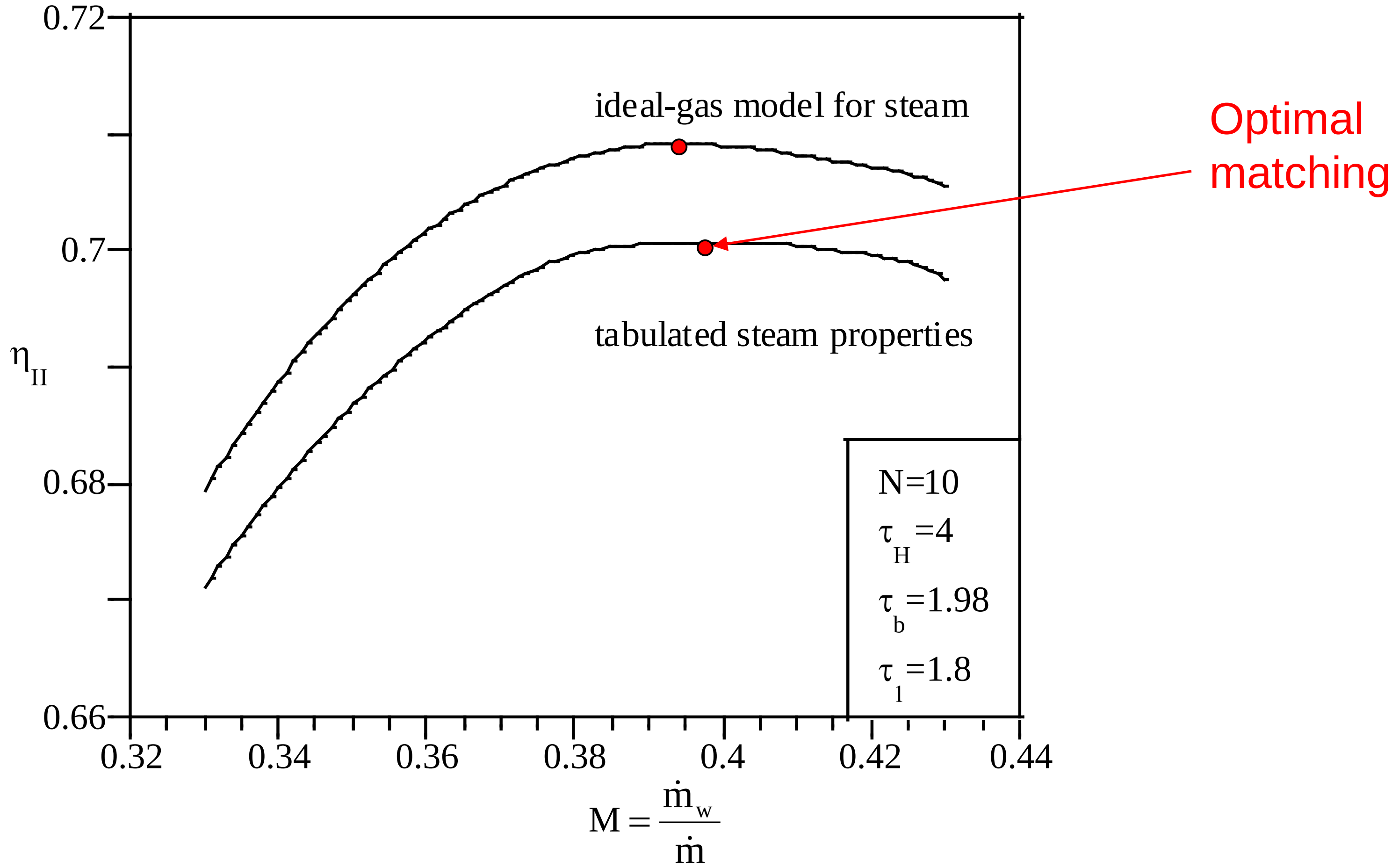
$$\dot{W} = \dot{m}e_{x,H} - \underbrace{T_0 \dot{S}_{gen}}_{\text{minimize}}$$

or directly using

$$\dot{W} = \dot{m}\left(e_{x,2} - e_{x,1}\right)$$

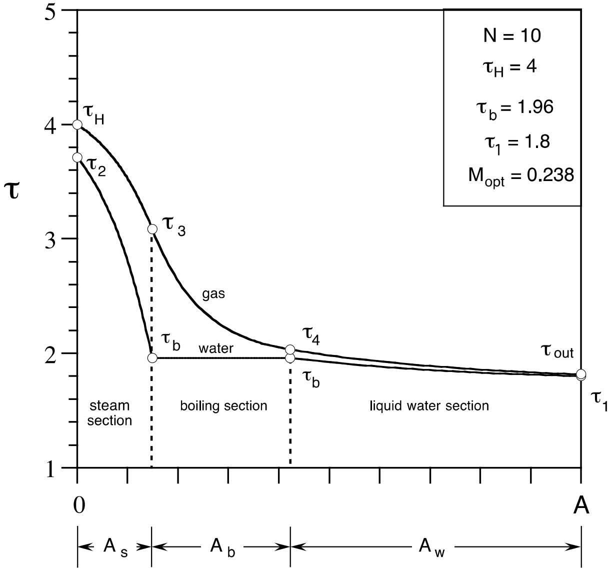
In dimensionless form:

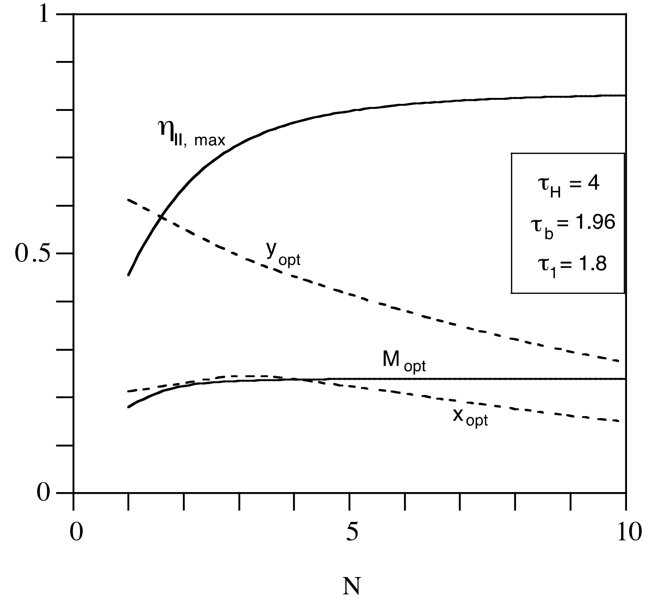
$$\eta_{II} = \frac{\dot{W}}{\dot{m}e_{x,H}} = \frac{\dot{m}_w\left(e_{x,2} - e_{x,1}\right)}{\dot{m}e_{x,H}}$$



Maximization of the second law efficiency by selecting the mass flow rate of the water stream

Concluding remarks:





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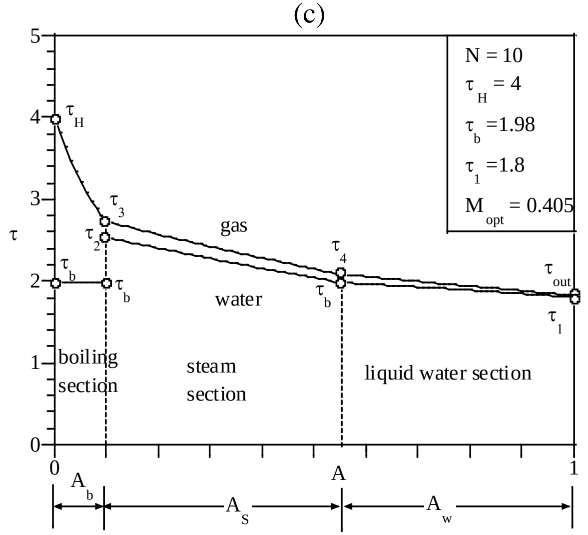
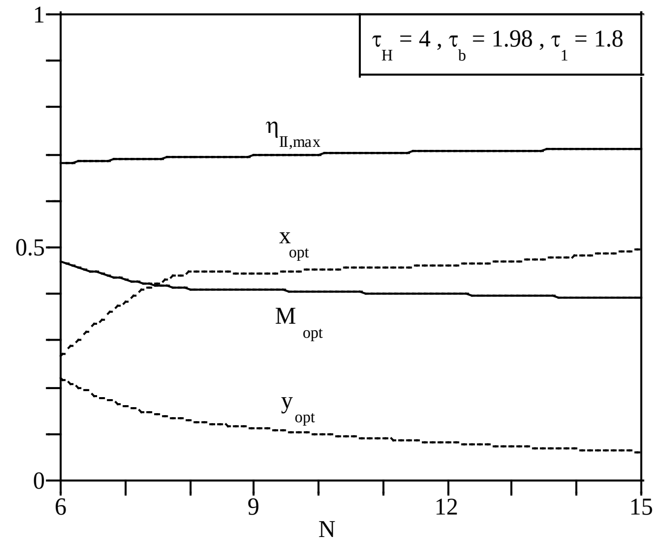
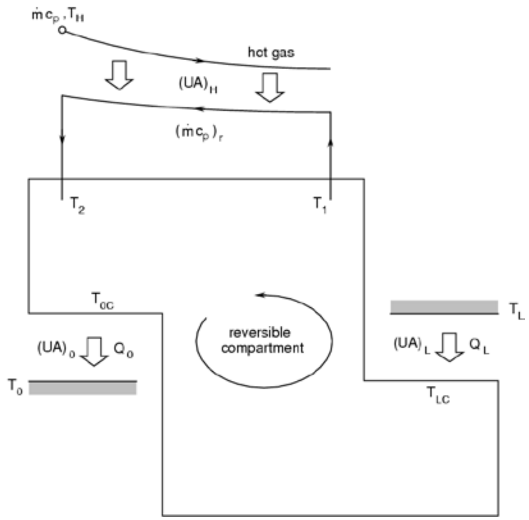
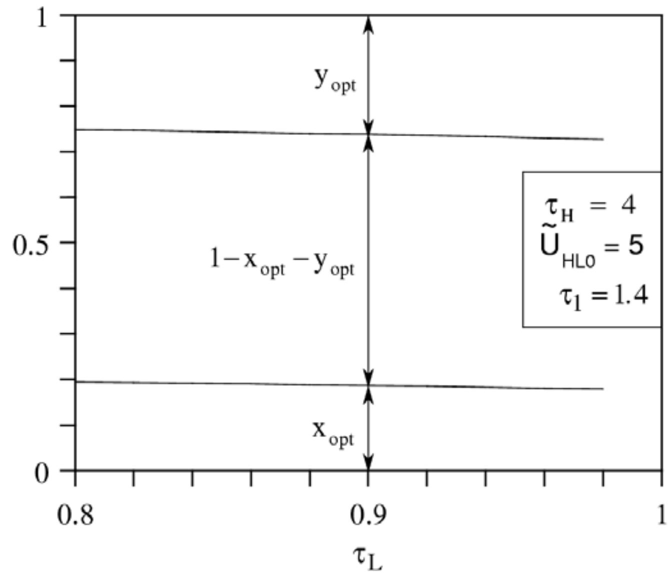




Optimal ratio is robust with respect to total surface area

J.C. Ordóñez





Optimal area allocation is robust with respect to refrigeration temperature